Thursday, November 13, 2003
Chapter 8, Problem 69.

Hot water at 90°C enters a 15-m section of a cast iron pipe \((k = 52 \text{ W/m} \cdot \text{°C})\) whose inner and outer diameters are 4 and 4.6 cm, respectively, at an average velocity of 0.8 m/s. The outer surface of the pipe, whose emissivity is 0.7, is exposed to the cold air at 10°C in a basement, with a convection heat transfer coefficient of 15 W/m²·°C. Taking the walls of the basement to be at 10°C also, determine (a) the rate of heat loss from the water and (b) the temperature at which the water leaves the basement.

Chapter 8, Solution 69

Hot water enters a cast iron pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

**Assumptions** 1 Steady flow conditions exist. 2 The inner surfaces of the pipe are smooth.

**Properties** We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-9)

\[
\begin{align*}
\rho &= 965.3 \text{ kg/m}^3; \\
k &= 0.675 \text{ W/m} \cdot \text{°C} \\
\nu &= \mu / \rho = 0.326 \times 10^{-6} \text{ m}^2 / \text{s}; \\
Pr &= 1.96 \\
C_p &= 4206 \text{ J/kg} \cdot \text{°C}
\end{align*}
\]

**Analysis** (a) The mass flow rate of water is

\[
\dot{m} = \rho A_c V = (965.3 \text{ kg/m}^3) \frac{\pi (0.04 \text{ m})^2}{4} (0.8 \text{ m/s}) = 0.9704 \text{ kg/s}
\]

The Reynolds number is

\[
\text{Re} = \frac{V m D_h}{\nu} = \frac{(0.8 \text{ m/s})(0.04 \text{ m})}{0.326 \times 10^{-6} \text{ m}^2 / \text{s}} = 98,062
\]

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

\[
L_h \approx L_f \approx 10D = 10(0.04 \text{ m}) = 0.4 \text{ m}
\]

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to \(\text{Re} = 98,062\) and \(\varepsilon / D = (0.026 \text{ cm})/(4 \text{ cm}) = 0.0065\) is determined from the Moody chart to be \(f = 0.034\). Then the Nusselt number becomes

\[
Nu = \frac{h D_h}{k} = 0.125 f \text{ Re Pr}^{1/3} = 0.125 \times 0.034 \times 98,062 \times 1.96^{1/3} = 521.6
\]

and

\[
h = \frac{k}{D_h} Nu = \frac{0.675 \text{ W/m} \cdot \text{°C}}{0.04 \text{ m}} (521.6) = 8801 \text{ W/m}^2 \cdot \text{°C}
\]

which is much greater than the convection heat transfer coefficient of 15 W/m²·°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the
inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of $90^\circ C$, and is determined to be

$$A_o = \pi d_0 L = \pi(0.046 \text{ m})(15 \text{ m}) = 2.168 \text{ m}^2$$

$$\dot{Q}_{\text{conv}} = h_o A_o (T_s - T_{\text{surr}}) = (15 \text{ W/m}^2 \cdot \circ C)(2.168 \text{ m}^2)(90 - 10)\circ C = 2601 \text{ W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon A_o \sigma (T_s^4 - T_{\text{surr}}^4) = (0.7)(2.168 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \circ K^4)[(90 + 273 \text{ K})^4 - (10 + 273 \text{ K})^4] = 942 \text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 2601 + 942 = 3543 \text{ W}$$

(b) The temperature at which water leaves the basement is

$$\dot{Q} = \dot{m} C_p (T_i - T_e) \Rightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m} C_p} = 90^\circ C - \frac{3543 \text{ W}}{(0.9704 \text{ kg/s})(4206 \text{ J/kg} \cdot ^\circ C)} = 89.1^\circ C$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{\text{pipe}} = \frac{\ln(D_2 / D_1)}{4\pi L} = \frac{\ln(4.6 / 4)}{4\pi (52 \text{ W/m}\cdot \circ C)(15 \text{ m})} = 1.65 \times 10^{-5} \circ C/\text{W}$$

$$\Delta T_{\text{pipe}} = \dot{Q}_{\text{total}} R_{\text{pipe}} = (3543 \text{ W})(1.65 \times 10^{-5} \circ C/\text{W}) = 0.06^\circ C$$

which justifies our assumption that the temperature drop across the pipe is negligible.
**PROBLEM 8.55**

KNOWN: Water flowing through a tube heated by cross-flow of a hot gas.

**FIND:** (a) The outlet temperature, $T_{ma}$, the average tube wall temperature, $T_w$, and the heat rate to the tube, $q$. (b) Compute and plot the outlet temperature, $T_{ma}$, as a function of gas velocity over the range $10 \leq V \leq 25$ m/s for water flow rates of 0.20, 0.25 and 0.30 kg/s; and (c) For fixed $T_{ma} = 40^\circ$C, compute and plot the water flow rate as a function of gas velocity for the range $10 \leq V \leq 25$ m/s for tube diameters of 50, 60 and 70 mm.

**SCHEMATIC:**

- $T_{in} = 300^\circ$C
- $V = 10$ m/s
- $T_{m,p} = 33^\circ$C
- $n = 0.25$ kg/s
- $L = 6$ m

**ASSUMPTIONS:** (1) Steady-state conditions. (2) Negligible kinetic and potential energy changes and axial conduction. (3) Fully developed flow and thermal conditions for internal flow. (4) Properties of the hot gas are those of atmospheric air. and (5) Negligible tube wall thermal resistance.

**PROPERTIES:** Table A.4, Water ($T_{in} = 300$ K): $p = 997$ kg/m$^3$, $c_p = 4179$ J/kg K, $\mu = 855 \times 10^{-6}$ N s/m$^2$, $k = 0.61$ W/m K, $Pr = 5.83$. Table A.4, Air ($T_{in} = 470$ K, 1 atm): $v = 32.39 \times 10^{-6}$ m$^2$/s, $k = 0.0333$ W/m K, $Pr = 0.666$.

**ANALYSIS:** (a) The water outlet temperature, $T_{ma}$, can be determined from Eq. 8.46a,

$$T_e - T_{in} = \frac{\pi D L}{8 n c_p} U$$

where the overall coefficient $U$ is

$$U = \frac{1}{h_e + h_w}$$

**Estimation of the internal flow coefficient, $h_e$:** Assuming $T_{in} = 380$ K for evaluation of water properties, characterize the flow,

$$Re_{in} = \frac{4m}{\pi D} = \frac{4 \times 0.25}{3.14 \times 0.0350} = 7446$$

while $2300 < Re_{in} < 10,000$, assume that the Dittus-Boelter correlation, Eq. 8.60, for turbulent flow is still appropriate,

$$\frac{h_{in}}{k} = 0.023 Re_{in}^{4/3} Pr^{1/3}$$

**Continued...**
PROBLEM 8.55 (Cont.)

\[ \dot{h}_c = 0.613 \text{ W/m} \cdot \text{K} \times 0.623(7746)^{0.81562} = 714.5 \text{ W/m}^2 \cdot \text{K} \]

**Estimation of external flow coefficient, \( \dot{h}_c \):** Assuming \( T_i = 450 \text{ K} \) for evaluation of (gas) air properties, characterize the flow,

\[ \text{Re}_{Da} = \frac{VD}{\nu} = 10 \text{ m/s} \times 0.050 \text{ m} = 500 \]

Using the Churchill-Bernstein correlation, Eq. 7.57, for cross flow over a cylinder.

\[ \text{Nu}_{Da} = 0.34 \left[ 1 + \left( \frac{0.62(15.437)^{0.81562})}{1 + (0.40)(0.686)^{2/3}} \right) \right]^{1/4} \]

\[ \dot{h}_c = \frac{\text{Nu}_{Da}k}{D} = \frac{282000}{0.050} = 567.5 \text{ W/m} \cdot \text{K} \]

From Eq. (2) with estimations for \( \dot{h}_c \) and \( \dot{h}_w \),

\[ U = \left( \frac{1704.5 \times 1.503}{1^3} \right) \text{ W/m}^2 \cdot \text{K} = 470 \text{ W/m}^2 \cdot \text{K} \]

and the outlet temperature from Eq. (1) is

\[ T_{out} = \frac{300 - T_{in}}{(300 - 23)^{0.25}} \times 470 \text{ W/m}^2 \cdot \text{K} = 34.5 \text{ K} \]

**Heat rate to the tube follows from the overall energy balance equation.**

\[ q = m_c \cdot c_p \cdot (T_{in} - T_{out}) \]

\[ q = 0.25 \text{ kg/s} \times 4179 \frac{\text{ J}}{\text{ kg} \cdot \text{K}} \times 34.5 \text{ K} = 12.0 \text{ kW} \]

Let's check the \( \bar{T}_w \) and \( \bar{T}_t \) assumptions. The water mean temperature based upon the \( T_{in} \), result is

\[ T_{in} = \frac{(T_{in} + T_{out})}{2} = 23 \text{ K} = 18.8 \text{ K} = 302 \text{ K} \]

and the average tube wall temperature follows from the thermal circuit,

\[ \frac{\bar{T}_t - T_i}{\bar{T}_w - T_i} = \frac{1}{h_i} + \frac{1}{h_o} \]

\[ 28.8 - \bar{T}_t \times 1/714.5 = 300 \]

\[ \bar{T}_t = 46.5 \text{ K} = 320 \text{ K} \]

Continued...
Thursday, November 13, 2003

Chapter 8, Problem 74.

Geothermal steam at 165°C condenses in the shell side of a heat exchanger over the tubes through which water flows. Water enters the 4-cm-diameter, 14-m-long tubes at 20°C at a rate of 0.8 kg/s. Determine the exit temperature of water and the rate of condensation of geothermal steam.

Chapter 8, Solution 74

Water is heated in a heat exchanger by the condensing geothermal steam. The exit temperature of water and the rate of condensation of geothermal steam are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The inner surfaces of the tube are smooth. 3 Air is an ideal gas with constant properties. 4 The surface temperature of the pipe is 165°C, which is the temperature at which the geothermal steam is condensing.

Properties The properties of water at the anticipated mean temperature of 85°C are (Table A-9)

\[ \rho = 968.1 \text{ kg/m}^3 \]
\[ k = 0.673 \text{ W/m.}^\circ \text{C} \]
\[ C_p = 4201 \text{ J/kg.}^\circ \text{C} \]
\[ Pr = 2.08 \]
\[ \nu = \frac{\mu}{\rho} = \frac{0.333 \times 10^{-3} \text{ kg/m.s}}{968.1 \text{ kg/m}^3} = 3.44 \times 10^{-7} \text{ m}^2/\text{s} \]
\[ h_{fg @165^\circ \text{C}} = 2066.5 \text{ kJ/kg} \]

Analysis The velocity of water and the Reynolds number are

\[ \dot{m} = \rho AV_m \longrightarrow 0.8 \text{ kg/s} = (968.1 \text{ kg/m}^3) \pi \frac{(0.04 \text{ m})^2}{4} V_m \longrightarrow V_m = 0.5676 \text{ m/s} \]
\[ \text{Re} = \frac{V_mD}{\nu} = \frac{(0.5676 \text{ m/s})(0.04 \text{ m})}{3.44 \times 10^{-7} \text{ m}^2/\text{s}} = 76,471 \]

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

\[ L_h \approx L_t \approx 10D = 10(0.04 \text{ m}) = 0.4 \text{ m} \]

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

\[ Nu = \frac{hD}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023(76,471)^{0.8}(2.08)^{0.4} = 248.7 \]

Heat transfer coefficient is
Thursday, November 13, 2003

\[ h = \frac{k}{D} \quad Nu = \frac{0.673 \text{ W/m} \cdot \text{°C}}{0.04 \text{ m}} \quad (248.7) = 4185 \text{ W/m}^2 \cdot \text{°C} \]

Next we determine the exit temperature of water,

\[ A_x = \pi DL = \pi (0.04 \text{ m})(14 \text{ m}) = 1.759 \text{ m}^2 \]

\[ T_e = T_s - (T_s - T_i)e^{-hD/(\pi C_p r)} = 165 - (165 - 20)e^{-\frac{(4185)(1.759)}{(0.5676)(4201)}} = 148.8^\circ\text{C} \]

The logarithmic mean temperature difference is

\[ \Delta T_{\text{in}} = \frac{T_x - T_i}{\ln\left(\frac{T_x}{T_i}\right)} = \frac{148.8 - 20}{\ln\left(\frac{165 - 148.8}{165 - 20}\right)} = 58.8^\circ\text{C} \]

The rate of heat loss from the exhaust gases can be expressed as

\[ \dot{Q} = hA_x \Delta T_{\text{in}} = (4185 \text{ W/m}^2 \cdot \text{°C})(1.759 \text{ m}^2)(58.8^\circ\text{C}) = 432,820 \text{ W} \]

The rate of condensation of steam is determined from

\[ \dot{Q} = \dot{m} h_{fg} \quad \rightarrow \quad 432.820 \text{ kW} = \dot{m}(2066.5 \text{ kJ/kg}) \quad \rightarrow \quad \dot{m} = 0.204 \text{ kg/s} \]
Chapter 11, Problem 21

The temperature of the filament of an incandescent lightbulb is 3200 K. Treating the filament as a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

Chapter 11, Solution 21

The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

**Assumptions** The filament behaves as a black body.

**Analysis** The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40 \, \mu m$ to $\lambda_2 = 0.76 \, \mu m$. Noting that $T = 3200 \, K$, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be

$$\lambda_1 T = (0.40 \, \mu m)(3200 \, K) = 1280 \, \mu m K \rightarrow f_{\lambda_1} = 0.0043964$$

$$\lambda_2 T = (0.76 \, \mu m)(3200 \, K) = 2432 \, \mu m K \rightarrow f_{\lambda_2} = 0.147114$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.14711424 - 0.0043964 = \textbf{0.142718} \quad \text{(or 14.3%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8 \, \mu m \cdot K \rightarrow \lambda_{\text{max power}} = \frac{2897.8 \, \mu m \cdot K}{3200 \, K} = \textbf{0.905 mm}$$
Chapter 11, Problem 41

The spectral emissivity function of an opaque surface at 1000 K is approximated as

\[
\varepsilon_\lambda = \begin{cases} 
0.4, & 0 \leq \lambda < 2 \mu m \\
0.7, & 2 \mu m \leq \lambda < 6 \mu m \\
0.3, & 6 \mu m \leq \lambda < \infty 
\end{cases}
\]

Determine the average emissivity of the surface and the rate of radiation emission from the surface, in W/m\(^2\).

Answers: 0.575, 32.6 kW/m\(^2\)

Chapter 11, Solution 41

The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

**Analysis** The average emissivity of the surface can be determined from

\[
\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_{\lambda_1}}(T)d\lambda + \varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_{\lambda_2}}(T)d\lambda + \varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_{\lambda_2}}(T)d\lambda}{\sigma T^4}
\]

\[
= \varepsilon_1 f_{0-\lambda_1} + \varepsilon_2 f_{\lambda_1-\lambda_2} + \varepsilon_3 f_{\lambda_2-\infty}
\]

\[
= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})
\]

where \(f_{\lambda_1}\) and \(f_{\lambda_2}\) are blackbody radiation functions corresponding to \(\lambda_1 T\) and \(\lambda_2 T\), determined from

\[
\lambda_1 T = (2 \mu m)(1000 K) = 2000 \mu mK \rightarrow f_{\lambda_1} = 0.066728
\]

\[
\lambda_2 T = (6 \mu m)(1000 K) = 6000 \mu mK \rightarrow f_{\lambda_2} = 0.737818
\]

\[
f_{0-\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1} \quad \text{since} \ f_0 = 0 \quad \text{and} \ f_{\lambda_1-\infty} = f_{\infty} - f_{\lambda_2} \quad \text{since} \ f_{\infty} = 1.
\]

And,

\[
\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = 0.575
\]

Then the emissive power of the surface becomes

\[
E = \varepsilon \sigma T^4 = 0.575(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1000 \text{ K})^4 = 32.6 \text{kW/m}^2
\]