New results on the non-linearity and stability of the estimation of the diffusion coefficient in a 2D elliptic equation

by

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We consider the problem of determining the diffusion coefficient $a(x)$ in a 2D elliptic equation from a distributed measurement $z$ in $H^1$ of the solution $u$ of the equation.

We shall first study the derivative of the $a \rightarrow u$ mapping, and give new conditions under which it is coercive.

Next, in order to study the non-linearity of the $a \rightarrow u$ mapping to be inverted, we shall compute a lower bound $R$ to the radii of curvature of the image curve in $H^1$, by the same mapping, of a segment $[a_0,a_1]$ joining two admissible parameters. The main finding is that, if one inverts for $b=1/a$, then the curvature $1/R$ of the image path goes to zero with $|\text{grad}(b_0-b_1)|/|b_0-b_1|$. This shows that the $b \rightarrow u$ mapping is less non-linear in the directions where the perturbation on the high frequencies of the diffusion parameter is small compared to the perturbation on the low frequencies.

This explains previous numerical results where a dramatic improvement in the performance of the inversion for $a$ was reported when a coarse-to-fine multiscale optimization was used. It suggests also the use of new regularization terms to stabilize the problem.

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