Abstract:

On a solution of some ill-posed problems by A.N.Tikhonov regularization method using MATLAB

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New numerical procedure based on A.N.Tikhonov regularization method [1, 2] was elaborated. It is known that regularization methods are widely used to resolve so-called ill-posed problems, including Fredholm integral equation of the first kind and systems of linear equations under bad conditions. A lot of numerical algorithms using regularization approach to solve such kind of problems was developed, including ones based on MATLAB platform [4]. But the central point of regularization method - the choice of optimal value of regularization parameter - is not resolved exhaustively, moreover, the common effective rule to obtain such parameter is unknown. Therefore many users have essential difficulties when they try to involve this technique into their computations.

The suggested approach, concerning this field, is based on the “Empirical Risk Minimisation” criterion [3]. The main peculiarity is the automatic determination of regularization parameter by “E-Structurezation” method (quasi-optimal), which does not require supplementary information about estimated data. It is of great importance because a level of the noise component of data in realistic applications usually is unknown. Thereby the use of method becomes very simple.

For example, let’s consider a common inverse ill-posed task:

\[ Fx = z \] (*)

where:

\( F \) - integral operator, \( z \) - observed value (probably containing stochastic mistake), \( x \) - the solution of inverse problem (must be obtained).

If matrix \( A \) - the kernel of operational equation (*),

REGILL.M (or REGILL.MEX) - file containing the regularization procedure, then the solution of (*) in MATLAB terms looks as:

\[ x = \text{regill}(A,z). \]

The proposed procedure allows us:

1. To solve ill-posed Fredholm integral equation of the first kind, including the restoration of derivation of the N order.

2. To solve linear equations systems under bad conditions, including the ones concerning to the restoration of linear and non-linear regression problems. In this case the method yields “Ridge Estimates” of regression coefficients.

3. To estimate a probability density distribution function (1,2, ... N dimensional). The procedure may be realised on MATLAB platform using standard MATLAB functions: SVD, “\" and FMINS; or like MEX function. Since 1991 my colleagues and me have used this procedure to solve wide range of ill-posed problems in the field of plasma physics. The special MATLAB toolbox based on Tikhonov regularization for calculation and modelling of plasma parameters (including Electron Energy Distribution Function) in discharges of plasma-chemical reactors was developed. In all cases we obtained reliable results by using REGILL.
References:


4. Regularization Tools.
   Version 2.0 16-March-93.
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