Want to do 2 things

Outline below

1) Write a code to solve 2-D conduction using isoparametric linear quadrilateral

2) Begin 2-D conduction: Solution of fluid problem

Laminar

a) Non-viscous flow
   - Stokke's flow
   - Low-speed flow, neglect advection terms
b) Viscous flow
   - Navier-Stokes Equations

Outline code for 2-D conduction
MAIN

1. Define node locations X, Y
2. Define connectivity KCOND
3. Define element candidate activity

function assemble global stiffness matrix (element by element)

function apply b.c.s.

Solve system of equations

Compute $K_{global}$

$NELE = \text{Length}(KCOND)$

$NNODES = \text{Length}(X)$

FOR $ELE = 1 : NELE$

$KC = \text{compute}_Kc(X_{local}, Y_{local}, KELE)$

$A = \text{assemble}(KC, \text{connect}(ELE))$

END

$K_{global} = A$

Apply B.C.s: Write a function specific for our problem

Type 1 conditions: - node number - value

Type 2 conditions: - element no - edge no. - value (cost or edge)
Type 3 condition:
- element no.
- edge no.
- h-value cast an edge
- Tref

Two steps:
1) Compute \( \text{h} \times \text{Tref} \) and add to
   RHTs just like a type 2
2) Compute \( K_{ij} \) and add to
   global stiffness matrix

Always do Type 4 condition last!!!

- \( \text{h} \), Too "Essential" BC

2) Stokes flow: pg 4385

2-D incompressible \((\rho = \text{const})\)

\[\n\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \hat{v} = u \hat{e}_x + v \hat{e}_y
\]

Continuity

\[\n\frac{\partial (\sigma_x - P)}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0
\]

\[\n\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial (\sigma_y - P)}{\partial y} = 0
\]

Constitutive relation to relate stress \((\sigma, \tau)\)

to velocity

\[\n\text{Newtonian Fluid} \quad \sigma_x = 2\eta \frac{\partial u}{\partial x}
\]
\[ \sigma_y = 2 \mu \frac{\partial u}{\partial y} \quad \mu = \text{kinematic viscosity} \]

\[ \tau_{xy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]

Two kinds of B.C.'s

\[ u = g(x,y) \quad v = h(x,y) \quad \text{on } S_1 \]

\[ \sigma_x = \text{surface traction} \]

\[ = (\sigma_x - P) n_x + \tau_{xy} n_y \quad \text{on } S_2 \]

\[ \sigma_y = \tau_{xy} n_x + (\sigma_y - P) n_y \]

\[ P = \text{thermodynamic pressure} \]

Start 9.3.3 on Friday

Use Galerkin MVR to get element equations —