Homework:
- Change \( X : [0.001 \; 0.001 \; 0.003, \; 0.007 \; 0.014 \; \ldots \text{etc.} \)
  So total length 0.1 m
- Change \( q_{gen} = 100,000 \; \text{W/m}^3 \)
  original soln

\[ \text{\text{- add } } T_{inf} = 20^\circ \text{C} \]

---

Natural BC's

\[
\begin{align*}
\int_0^1 \left\{ N \right\} \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx &= \text{by part 3} \\
&= k \frac{\partial T}{\partial x} \left. \left\{ N \right\} \right|_0^1 \\
&= \int_0^1 \left( k \frac{\partial T}{\partial x} \right) dx \\
&= \int_0^1 \frac{\partial T}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dx
\end{align*}
\]

Look at this term

Note \( N_1 = 1 \; \text{at } x=0 \; \text{and } N_2 = 0 \)

\( N_1 = 0 \; \text{at } x=le \; \text{and } N_2 = 1 \)

\[
\left\{ \begin{array}{c}
- k \frac{dT}{dx} \bigg|_0 \\
+ k \frac{dT}{dx} \bigg|_{le}
\end{array} \right\} = \text{"Natural BCs"} = \left\{ \begin{array}{c}
g_{in} \bigg|_0 \\
g_{in} \bigg|_{le}
\end{array} \right\}
\]
Suppose \( q_x = 100 \text{ W/m}^2 \)

\[ q_{X=0} \]

In the assembled system, these natural BC's can only affect nodes on the boundary. In the 1-D case, node 1 and node n nodes

\[
\text{RHS} = \begin{cases} 
9_{x=0} \\
0 \\
0 \\
\vdots \\
q_{x=L} \\
q_{x=0} \\
0 
\end{cases}
\]

due to natural BC's

Note also get contributions to RHS from \( R_a \) and \( R_n \)

\[
\text{RHS} = \{ R_a \} + \{ R_n \} + \text{No. B.C.'s}
\]

Also \( K_{e_1} = K_c + K_n \)

\( A \) assemble into "A"
Original Equation
\[
\frac{d}{dx} \left( k \frac{dT}{dx} \right) + Q'' = \frac{\hbar P}{A} (T - T_0)
\]
Can write as
\[
A \frac{d}{dx} \left( k \frac{dT}{dx} \right) + A \cdot Q'' = \hbar P (T - T_0)
\]
then get matrices exactly as book (8.21)
Makes it easier to account for heat loss at fin tip by applying 8.15a)

\[
\mathbf{R}_T = - \int (\mathbf{\hat{q}} \cdot \mathbf{\hat{n}}) d\mathbf{N} \quad d\mathbf{N}
\]

\[
- \mathbf{\hat{q}} \cdot \mathbf{\hat{n}} \quad \text{is heat flow into domain}
\]
\[
= \int_{\text{in}} q'' \{N\} \, d\mathbf{N}
\]
For heat gain at tip \( q'' = \hbar (T_0 - T) \)

\[
\text{integral} = \int_{\text{in}} h(T_0 - T) \{N\} \, d\mathbf{N}
\]
\[
\text{integral} = \int_{\text{in}} hT_0 \{N\} \, d\mathbf{N} - \int_{\text{out}} hT_0 \{N\} \, d\mathbf{N}
\]
On \( A = L \) (the right end of \( D \))

\[
[SN] = \{10\} \quad [LN] = \{10\}
\]

\[
T(0) = 1000 \quad \text{Type I or Essential}
\]

\[
\text{BC's} = T_1 = 100 \quad \text{or diagonal}
\]

Modify [\( A \)]\{1\} = \{0.3\}

\[
\text{A}(1,1) = \frac{0.3}{4} \quad \text{A}(1) = \{0.3\}
\]

\[
\text{RHS}(1) = \{000\}
\]