Did v MWR for F.E.M.
Galerkin
1) choose interpolating function
2) arrange so the field values (T, u, etc.) are the unknown coef. 
   \[ T^i(x) = \sum \lambda_i N_i(x) \]
3) formulate the MWR statements. 
   Let \( W_i(x) = \frac{\partial T^i(x)}{\partial T_i} = N_i(x) \)
4) use integration by parts to "level" the order of differentiation in integrals. 
   (get derivatives as \( N_i \) and \( T \) as close as possible)
5) write the "n" equations in a matrix form 
6) evaluate integrals.

From our example (lost time)
\[ \frac{d^2 T}{dx^2} - T = 0 \]
\( \Theta \) MWR gives
\[
\begin{bmatrix}
-1 & 1 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
e \\
e
\end{bmatrix}
- \frac{\Delta x}{6}
\begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
\frac{\Delta T^e}{\Delta x} \\
\frac{\Delta T^e}{\Delta x}
\end{bmatrix}
\]

\[
\frac{dT^e}{dx} = 0
\]

**IIa**

From integration by parts:

\[
\frac{dT^e}{dx} \bigg|_0^{le} = \left\{ \frac{dT^e}{dx} N_i \bigg|_0^{le} \right\} = \left\{ \frac{dT^e}{dx} N_i \bigg|_0^{le} \right\} = \left\{ \frac{dT^e}{dx} \bigg|_0^{le} \right\} - \left\{ \frac{dT^e}{dx} \bigg|_0^{le} \right\}
\]

\[
= \left\{ -\frac{dT^e}{dx} \bigg|_0^{le} \right\} + \left\{ \frac{dT^e}{dx} \bigg|_0^{le} \right\}
\]

\[
= \left\{ \frac{dT^e}{dx} \bigg|_0^{le} \right\}
\]

The "natural" boundary conditions:

\[
B.C.'s
\]

\[
\begin{array}{c}
N(x) \\
N(0) \\
N(L)
\end{array}
\]

\[
\begin{array}{c}
x=0 \\
le
\end{array}
\]

\[
\begin{array}{c}
x=L
\end{array}
\]

If no conditions specified, then solve as:

\[
\frac{dT^e}{dx} \bigg|_0^{le} = 0
\]

for element 5.
We have the element level equation.

(3) Assemble the equations.

Look at simple Domain:

\[\begin{array}{c@{}c@{}c@{}c@{}c}
1 & 2 & 2 & 3 & 3 & 4 \\
\end{array}\]  \rightarrow \text{global node numbers}

\[\begin{array}{c@{}c}
0 & 1 \\
2 & 3 \\
\end{array}\]  \rightarrow \text{local node numbers}

For a general F.E.M. problem, the connectivity matrix defines the global d.o.f. (node numbers) corresponding to local ones.

Connectivity matrix

\[
\begin{bmatrix}
1 & 2 \\
2 & 3 \\
3 & 4 \\
\end{bmatrix}
\]

Entries are global node numbers, rows are elements, cols are local node numbers.
Another connectivity matrix example

2-D element

Domain

10 elements
18 nodes

Connectivity matrix

<table>
<thead>
<tr>
<th>Row</th>
<th>1 2 5 4</th>
<th>- ele 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3 6 5</td>
<td>ele 2</td>
</tr>
<tr>
<td>4</td>
<td>5 8 7</td>
<td>ele 3</td>
</tr>
<tr>
<td>5</td>
<td>6 9 8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8 12 11</td>
<td></td>
</tr>
</tbody>
</table>

Back to 1-D example

Assemble equ's by hand
and also w/ connectivity