\[
\frac{1}{L} \frac{d}{dx} \left( Pe \cdot T \right) = \frac{d^2 T}{dx^2}
\]
\[T(0) = 1\]
\[T(1) = 0\]

\[
\frac{1}{L} \frac{d}{dx} \left( S \cdot T \right) = \frac{h}{c_p} \frac{d}{dx} \left( \frac{dT}{dx} \right)
\]

\[
\frac{d^2 u}{dx^2} + u = 1
\]
\[u(0) = 1\]
\[u(1) = 0\]

Let's look at HW prob #2:
\[\tilde{u}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3\]

Find \(a_0, a_1, a_2, a_3\) so that \(\tilde{u}(x)\) is a MPW approximation to the solution of Problem 2.

First: Find values for two of constants in terms of all others so BCs are satisfied.
\( \tilde{u}(0) = 1 = a_0 \)

\[ \tilde{u}(1) = a_0 + a_1 + a_2 + a_3 = 0 \]

\[ a_1 = -\frac{1}{1 - a_2 - a_3} = -(1 + a_2 + a_3) \]

\[ \tilde{u}_r(x) = 1 + (-1 - a_2 - a_3)x + a_2 x^2 + a_3 x^3 \]

\[ = 1 - x + a_2 (x^2 - x) + a_3 (x^3 - x) \]

Use 4 different MWR methods to find \( a_2, a_3 \)

1. Collocation at \( x_1 = \frac{1}{3}, x_2 = \frac{2}{3} \)

\[ R(x) = ? \]

\[ \frac{d^2 u}{dx^2} = 2a_2 + 6a_3 x \]

\[ R(x) = (2a_2 + 6a_3 x) + (1 - x + a_2 (x^2 - x) + a_3 (x^3 - x)) - x \]

\[ = 2a_2 - x + a_2 (x^2 - x + 2) + a_3 (x^3 - 3x) \]

\[ R(x_1 = \frac{1}{3}) = 0 = -\frac{1}{3} + a_2 \left( \frac{1}{3} - \frac{1}{3} + 2 \right) + a_3 \left( \frac{1}{27} + \frac{5}{3} \right) \]

\[ R(x_2 = \frac{2}{3}) = 0 = -\frac{2}{3} + a_2 \left( \frac{4}{9} - \frac{2}{3} + 2 \right) + a_3 \left( \frac{8}{27} + \frac{5}{3} \right) \]
\[ A \{ a \} = \{ b \} \]
\[ a_2 = 0.02163 \]
\[ a_3 = 0.1730769 \]
\[ a_6 = 1.0 \]
\[ a_4 = -1.19471 \]