Chapter 8, Solution 24.

Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. The surface temperature of the pipe is constant.
3. The thermal resistance of the pipe is negligible.
4. Air properties are to be used for exhaust gases.

**Properties** The properties of air at the average temperature of \(\frac{250+150}{2}=200^\circ\text{C}\) are (Table A-15)

\[
c_p = 1023 \text{ J/kg.}^\circ\text{C} \\
R = 0.287 \text{ kJ/kg.}^\circ\text{K}
\]

Also, the heat of vaporization of water at 1 atm or \(100^\circ\text{C}\) is \(h_{fg} = 2257 \text{ kJ/kg}\) (Table A-9).

**Analysis** The density of air at the inlet and the mass flow rate of exhaust gases are

\[
\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.}^\circ\text{K})(250+273 \text{ K})} = 0.7662 \text{ kg/m}^3
\]

\[
\dot{m} = \rho A_c V_{\text{avg}} = \rho \left( \frac{\pi D^2}{4} \right) V_{\text{avg}} = (0.7662 \text{ kg/m}^3) \frac{\pi(0.03 \text{ m})^2}{4} (5 \text{ m/s}) = 0.002708 \text{ kg/s}
\]

The rate of heat transfer is

\[
\dot{Q} = \dot{m} c_p (T_i - T_e) = (0.002708 \text{ kg/s})(1023 \text{ J/kg.}^\circ\text{C})(250 - 150^\circ\text{C}) = 277.0 \text{ W}
\]

The logarithmic mean temperature difference and the surface area are

\[
\Delta T_{\text{in}} = \frac{T_e - T_i}{\ln \left( \frac{T_e - T_i}{T_e - T} \right)} = \frac{150 - 250}{\ln \left( \frac{110 - 150}{110 - 250} \right)} = 79.82^\circ\text{C}
\]

\[
\dot{Q} = h A_s A \Delta T_{\text{in}} \\
\dot{Q} = \frac{277.0 \text{ W}}{(120 \text{ W/m}^2.\text{}^\circ\text{C})(79.82^\circ\text{C})} = 0.02891 \text{ m}^2
\]

Then the tube length becomes

\[
A_s = \pi D L \\
A_s = \frac{0.02891 \text{ m}^2}{\pi(0.03 \text{ m})} = 0.3067 \text{ m} = 30.7 \text{ cm}
\]

The rate of evaporation of water is determined from

\[
\dot{Q} = \dot{m}_{\text{evap}} h_{fg} \\
\dot{m}_{\text{evap}} = \frac{\dot{Q}}{h_{fg}} = \frac{(0.2770 \text{ kW})}{(2257 \text{ kJ/kg})} = 0.0001227 \text{ kg/s} = 0.442 \text{ kg/h}
\]
Chapter 8, Solution 26C.

The friction factor for flow in a tube is proportional to the pressure drop. Since the pressure drop along the flow is directly related to the power requirements of the pump to maintain flow, the friction factor is also proportional to the power requirements. The applicable relations are

\[ \Delta P = f \frac{L}{D} \frac{\rho V^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \frac{\dot{m} \Delta P}{\rho} \]

Chapter 8, Solution 27C.

The shear stress at the center of a circular tube during fully developed laminar flow is zero since the shear stress is proportional to the velocity gradient, which is zero at the tube center.

Chapter 8, Solution 28C.

Yes, the shear stress at the surface of a tube during fully developed turbulent flow is maximum since the shear stress is proportional to the velocity gradient, which is maximum at the tube surface.

Chapter 8, Solution 29C.

In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, the pressure drop will also double (the pressure drop is proportional to length).

Chapter 8, Solution 30C.

Yes, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2 since \( \dot{V} = \dot{V}_{\text{avg}} A_c = (V_{\text{max}} / 2) A_c \).

Chapter 8, Solution 31C.

No, the average velocity in a circular pipe in fully developed laminar flow cannot be determined by simply measuring the velocity at \( R/2 \) (midway between the wall surface and the centerline). The mean velocity is \( V_{\text{max}}/2 \), but the velocity at \( R/2 \) is

\[ V(R/2) = V_{\text{max}} \left( 1 - \frac{r^2}{R^2} \right)_{r=R/2} = \frac{3V_{\text{max}}}{4} \]

Chapter 8, Solution 32C.
In fully developed laminar flow in a circular pipe, the pressure drop is given by
\[ \Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2} \]

The mean velocity can be expressed in terms of the flow rate as
\[ V_{\text{avg}} = \frac{\hat{V}}{A_c} = \frac{\hat{V}}{\pi D^2 / 4}. \]

Substituting,
\[ \Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2} = \frac{32\mu L}{D^2} \frac{\hat{V}}{\pi D^2 / 4} = \frac{128\mu L \hat{V}}{\pi D^4} \]

Therefore, at constant flow rate and pipe length, the pressure drop is inversely proportional to the 4th power of diameter, and thus reducing the pipe diameter by half will increase the pressure drop by a factor of 16.

**Chapter 8, Solution 33C.**

In fully developed laminar flow in a circular pipe, the pressure drop is given by
\[ \Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2} \]

When the flow rate and thus mean velocity are held constant, the pressure drop becomes proportional to viscosity. Therefore, pressure drop will be reduced by half when the viscosity is reduced by half.

**Chapter 8, Solution 34C.**

The tubes with rough surfaces have much higher heat transfer coefficients than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the heat transfer coefficient is negligible.

**Chapter 8, Solution 39.**

The convection heat transfer coefficients for the flow of air and water are to be determined under similar conditions.

*Assumptions* 1 Steady flow conditions exist. 2 The surface heat flux is uniform. 3 The inner surfaces of the tube are smooth.

*Properties* The properties of air at 25°C are (Table A-15)
\[ k = 0.02551 \text{ W/m.}^°\text{C} \]
\[ \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \]
\[ \text{Pr} = 0.7296 \]

The properties of water at 25°C are (Table A-9)
\[ \rho = 997 \text{ kg/m}^3 \]
\[ k = 0.607 \text{ W/m.}^\circ\text{C} \]
\[ \nu = \frac{\mu}{\rho} = 0.891 \times 10^{-3} / 997 = 8.937 \times 10^{-7} \text{ m}^2/\text{s} \]
\[ \text{Pr} = 6.14 \]

**Analysis** The Reynolds number is
\[
\text{Re} = \frac{VD}{\nu} = \frac{(2 \text{ m/s})(0.08 \text{ m})}{1.562 \times 10^{-3} \text{ m}^2/\text{s}} = 10,243
\]

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly
\[
L_h \approx L_t \approx 10D = 10(0.08 \text{ m}) = 0.8 \text{ m}
\]

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from
\[
\text{Nu} = \frac{hD}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(10,243)^{0.8}(0.7296)^{0.4} = 32.76
\]

Heat transfer coefficient is
\[
h = \frac{k}{D} \text{Nu} = \frac{0.02551 \text{ W/m.}^\circ\text{C}}{0.08 \text{ m}} (32.76) = 10.45 \text{ W/m}^2.\circ\text{C}
\]

Repeating calculations for water:
\[
\text{Re} = \frac{VD}{\nu} = \frac{(2 \text{ m/s})(0.08 \text{ m})}{8.937 \times 10^{-7} \text{ m}^2/\text{s}} = 179,035
\]
\[
\text{Nu} = \frac{hD}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(179,035)^{0.8}(6.14)^{0.4} = 757.4
\]
\[
h = \frac{k}{D} \text{Nu} = \frac{0.607 \text{ W/m.}^\circ\text{C}}{0.08 \text{ m}} (757.4) = 5747 \text{ W/m}^2.\circ\text{C}
\]

**Discussion** The heat transfer coefficient for water is 550 times that of air.

**Chapter 8, Solution 40.**

Air flows in a pipe whose inner surface is not smooth. The rate of heat transfer is to be determined using two different Nusselt number relations.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas with constant properties. 3 The pressure of air is 1 atm.

**Properties** Assuming a bulk mean fluid temperature of 20°C, the properties of air are (Table A-15)
\[
\rho = 1.204 \text{ kg/m}^3
\]
\[ k = 0.02514 \text{ W/m.}^\circ\text{C} \]
\[ \nu = 1.516 \times 10^{-5} \text{ m}^2/\text{s} \]
\[ c_p = 1007 \text{ J/kg.}^\circ\text{C} \]
\[ \text{Pr} = 0.7309 \]
**Analysis** The mean velocity of air and the Reynolds number are

\[
V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.065 \text{ kg/s}}{(1.204 \text{ kg/m}^3)\pi(0.12 \text{ m})^2 / 4} = 4.773 \text{ m/s}
\]

\[
\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(4.773 \text{ m/s})(0.12 \text{ m})}{1.516 \times 10^{-5} \text{ m}^2/\text{s}} = 37,785
\]

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

\[
L_h \approx L_t = 10 D = 10(0.12 \text{ m}) = 1.2 \text{ m}
\]

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire duct. The friction factor may be determined from Colebrook equation using EES to be

\[
\frac{1}{\sqrt{f}} = -2\log \left( \frac{e / D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \longrightarrow \frac{1}{\sqrt{f}} = -2\log \left( \frac{0.00022 / 0.12}{3.7} + \frac{2.51}{37,785\sqrt{f}} \right) \longrightarrow f = 0.02695
\]

The Nusselt number from Eq. 8-66 is

\[
Nu = 0.125 f \text{RePr}^{1/3} = 0.125(0.02695)(37,785)(0.7309)^{1/3} = 114.7
\]

Heat transfer coefficient is

\[
h = \frac{k}{(D\pi)(0.12 \text{ m})} = 24.02 \text{ W/m}^2\cdot\text{°C}
\]

Next we determine the exit temperature of air

\[
A = \pi D L = \pi(0.12 \text{ m})(5 \text{ m}) = 1.885 \text{ m}^2
\]

\[
T_e = T_x - (T_x - T_i)e^{-hA/(\dot{m} c_p)} = 50 - (50 - 10)e^{-\frac{(24.02)(1.885)}{(0.065)(1007)}} = 30.0^\circ\text{C}
\]

This result verifies our assumption of bulk mean fluid temperature that we used for property evaluation. Then the rate of heat transfer becomes

\[
\dot{Q} = \dot{m} c_p(T_e - T_i) = (0.065 \text{ kg/s})(1007 \text{ J/kg} \cdot \text{°C})(30.0 - 10)\text{°C} = 1307 \text{ W}
\]

Repeating the calculations using the Nusselt number from Eq. 8-70:

\[
Nu = \frac{(f / 8)(\text{Re} - 1000) \text{Pr}^{1/3}}{1 + 12.7(f / 8)^{0.5} (\text{Pr}^{2/3} - 1)} = \frac{(0.02695 / 8)(37,785 - 1000)(0.7309)}{1 + 12.7(0.02695 / 8)^{0.5} (0.7309^{2/3} - 1)} = 105.2
\]

\[
h = \frac{k}{(D\pi)\text{Nu}} = \frac{0.02514 \text{ W/m} \cdot \text{°C}}{(0.12 \text{ m})} = 22.04 \text{ W/m}^2\cdot\text{°C}
\]

\[
T_e = T_x - (T_x - T_i)e^{-hA/(\dot{m} c_p)} = 50 - (50 - 10)e^{-\frac{(22.04)(1.885)}{(0.065)(1007)}} = 28.8^\circ\text{C}
\]

\[
\dot{Q} = \dot{m} c_p(T_e - T_i) = (0.065 \text{ kg/s})(1007 \text{ J/kg} \cdot \text{°C})(28.8 - 10)\text{°C} = 1230 \text{ W}
\]

The result by Eq. 8-66 is about 6 percent greater than that by Eq. 8-70.