A furnace is shaped like a long semicylindrical duct of diameter $D = 15$ ft. The base and the dome of the furnace have emissivities of 0.5 and 0.9 and are maintained at uniform temperatures of 550 and 1800 R, respectively. Determine the net rate of radiation heat transfer from the dome to the base surface per unit length during steady operation.

Chapter 12, Solution 32

The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

**Assumptions**

1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**

The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$.

**Analysis**

The view factor from the base to the dome is first determined from

\[ F_{11} = 0 \quad \text{(flat surface)} \]

\[ F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)} \]

The net rate of radiation heat transfer from dome to the base surface can be determined from

\[ \dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^4 - T_2^4)}{1 - \varepsilon_1 + \frac{1}{A_1 F_{11}} + \frac{1 - \varepsilon_2}{A_2 F_{12}}} = -\frac{(0.1714 \times 10^{-8} \, \text{Btu/h-ft}^2 \cdot \text{R}^4) \times [(550 \, \text{R})^4 - (1800 \, \text{R})^4]}{1 - 0.5 + \frac{1}{(15 \, \text{ft}^2)(0.5)} + \frac{1 - 0.9}{(15 \, \text{ft}^2)(1)} + \frac{\pi(15 \, \text{ft})(1 \, \text{ft})}{2}(0.9)} \]

\[ = 1.311 \times 10^6 \, \text{Btu/h} \]

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.
Two parallel disks of diameter $D = 0.6$ m separated by $L = 0.4$ m are located directly on top of each other. Both disks are black and are maintained at a temperature of 700 K. The back sides of the disks are insulated, and the environment that the disks are in can be considered to be a blackbody at $T_\infty = 300$ K. Determine the net rate of radiation heat transfer from the disks to the environment. Answer: 5505 W

Chapter 12, Solution 33

Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**
The emissivities of all surfaces are $\varepsilon = 1$ since they are black.

**Analysis**
Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 12-7, we read

\[ F_{12} = F_{21} = 0.26 \]
\[ F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)} \]

The net rate of radiation heat transfer from the disks into the environment then becomes

\[ \dot{Q}_3 = \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13} \]
\[ \dot{Q}_3 = 2F_{13}A_1\sigma(T_1^4 - T_3^4) \]
\[ = 2(0.74)(6\pi(0.3 \text{ m})^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(700 \text{ K})^4 - (300 \text{ K})^4] \]
\[ = 5505 \text{ W} \]
A furnace is shaped like a long equilateral-triangular duct where the width of each side is 2 m. Heat is supplied from the base surface, whose emissivity is $\varepsilon_1 = 0.8$, at a rate of 800 W/m\(^2\) while the side surfaces, whose emissivities are 0.5, are maintained at 500 K. Neglecting the end effects, determine the temperature of the base surface. Can you treat this geometry as a two-surface enclosure?

Chapter 12, Solution 34

A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.
4. End effects are neglected.

**Properties**
The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$.

**Analysis**
This geometry can be treated as a two-surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from

$$
\dot{Q}_{12} = \frac{\sigma (T_1^4 - T_2^4)}{A_1 \varepsilon_1 + A_2 \varepsilon_2}
$$

$$
800 \text{ W} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(T_1^4 - (500 \text{ K})^4)}{1 - 0.8 + 1 - 0.5}
$$

$$
\Rightarrow T_1 = 543 \text{ K}
$$

Note that $A_1 = 1 \text{ m}^2$ and $A_2 = 2 \text{ m}^2$. 
A spherical tank of diameter \( D = 2 \) m that is filled with liquid nitrogen at 100 K is kept in an evacuated cubic enclosure whose sides are 3 m long. The emissivities of the spherical tank and the enclosure are \( \varepsilon_1 = 0.1 \) and \( \varepsilon_2 = 0.8 \), respectively. If the temperature of the cubic enclosure is measured to be 240 K, determine the net rate of radiation heat transfer to the liquid nitrogen. Answer: 228 W

**Chapter 12, Solution 38**

A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.
4. The thermal resistance of the tank is negligible.

**Properties**
The emissivities of surfaces are given to be \( \varepsilon_1 = 0.1 \) and \( \varepsilon_2 = 0.8 \).

**Analysis**
We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that \( F_{12} = 1 \), for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from

\[
\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma A_2}{1 - \varepsilon_2} \left( T_1^4 - T_2^4 \right)
\]

\[
= -\sigma \frac{\pi D^2}{6} \frac{8.0 \times 10^{-8}}{0.1} \left( 100^4 - 240^4 \right)
\]

\[
= 228 \text{ W}
\]
Consider a circular grill whose diameter is 0.3 m. The bottom of the grill is covered with hot coal bricks at 1100 K, while the wire mesh on top of the grill is covered with steaks initially at 5 °C. The distance between the coal bricks and the steaks is 0.20 m. Treating both the steaks and the coal bricks as blackbodies, determine the initial rate of radiation heat transfer from the coal bricks to the steaks. Also, determine the initial rate of radiation heat transfer to the steaks if the side opening of the grill is covered by aluminum foil, which can be approximated as a reradiating surface. Answers: 1674 W, 3757 W

Chapter 12, Solution 41

A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**
The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.

**Analysis**
We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 12-1),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15}{0.20} = 0.75$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2} = 1 + \frac{0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{2j} = \frac{1}{2} \left[ S - 4 \left( \frac{R_j}{R_i} \right) \right]^{1/2} = \frac{1}{2} \left[ 3.7778 - 4 \left( \frac{0.75}{0.75} \right) \right]^{1/2} = 0.2864$$

(It can also be determined from Fig. 12-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$Q_{12} = F_{12} A \sigma (T_1^4 - T_2^4)$$

$$= (0.2864) \left[ \frac{\pi (0.3 m)^2}{4} \right] \left[ 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \right] \left[ (1100 \text{ K})^4 - (278 \text{ K})^4 \right]$$

$$= 1674 \text{ W}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a
reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the room can be determined from

\[
\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}
\]

where

\[
E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1100 \text{ K})^4 = 83,015 \text{ W/m}^2
\]

\[
E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(18 + 273 \text{ K})^4 = 407 \text{ W/m}^2
\]

and

\[
A_1 = A_2 = \frac{\pi(0.3 \text{ m})^2}{4} = 0.07069 \text{ m}^2
\]

\[
R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(0.07069 \text{ m}^2)(0.2864)} = 49.39 \text{ m}^2
\]

\[
R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(0.07069 \text{ m}^2)(1 - 0.2864)} = 19.82 \text{ m}^2
\]

Substituting,

\[
\dot{Q}_1 = \frac{(83,015 - 407) \text{ W/m}^2}{\left(\frac{1}{49.39 \text{ m}^2} + \frac{1}{2(19.82 \text{ m}^2)}\right)^{-1}} = 3757 \text{ W}
\]
A thin aluminum sheet with an emissivity of 0.15 on both sides is placed between two very large parallel plates, which are maintained at uniform temperatures \( T_1 = 900 \text{ K} \) and \( T_2 = 650 \text{ K} \) and have emissivities \( \varepsilon_1 = 0.5 \) and \( \varepsilon_2 = 0.8 \), respectively. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and compare the result with that without the shield.

**Chapter 12, Solution 51**

A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

**Assumptions**
1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**

The emissivities of surfaces are given to be \( \varepsilon_1 = 0.5 \), \( \varepsilon_2 = 0.8 \), and \( \varepsilon_3 = 0.15 \).

**Analysis**

The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is

\[
\dot{Q}_{12, \text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_3,1} + \frac{1}{\varepsilon_3,2} - 1\right)}
\]

\[
= \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(900 \text{ K})^4 - (650 \text{ K})^4]}{(1 + \frac{1}{0.5} - 1) + \left(\frac{1}{0.15} + \frac{1}{0.15} - 1\right)}
\]

\[
= 1857 \text{ W/m}^2
\]

The net rate of radiation heat transfer between the plates in the case of no shield is

\[
\dot{Q}_{12, \text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(900 \text{ K})^4 - (650 \text{ K})^4]}{(1 + \frac{1}{0.5} - 1)} = 12,035 \text{ W/m}^2
\]

Then the ratio of radiation heat transfer for the two cases becomes

\[
\frac{\dot{Q}_{12, \text{one shield}}}{\dot{Q}_{12, \text{no shield}}} = \frac{1857 \text{ W}}{12,035 \text{ W}} \approx \frac{1}{6}
\]
Two very large parallel plates are maintained at uniform temperatures of \( T_1 = 1000 \) K and \( T_2 = 800 \) K and have emissivities of \( \varepsilon_1 = \varepsilon_2 = 0.2 \), respectively. It is desired to reduce the net rate of radiation heat transfer between the two plates to one-fifth by placing thin aluminum sheets with an emissivity of 0.15 on both sides between the plates. Determine the number of sheets that need to be inserted.

**Chapter 12, Solution 53**

Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

**Assumptions**

1. Steady operating conditions exist
2. The surfaces are opaque, diffuse, and gray.
3. Convection heat transfer is not considered.

**Properties**

The emissivities of surfaces are given to be \( \varepsilon_1 = 0.2 \), \( \varepsilon_2 = 0.2 \), and \( \varepsilon_3 = 0.15 \).

**Analysis**

The net rate of radiation heat transfer between the plates in the case of no shield is

\[
\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)} = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(1000 \text{ K})^4 - (800 \text{ K})^4] \\
= \left(\frac{1}{0.2} + \frac{1}{0.2} - 1\right) \\
= 3720 \text{ W/m}^2
\]

The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to one-fifth can be determined from

\[
\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + N_{\text{shields}} \left(\frac{1}{\varepsilon_3,1} + \frac{1}{\varepsilon_3,2} - 1\right)}
\]

\[
\frac{1}{5}(3720 \text{ W/m}^2) = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(1000 \text{ K})^4 - (800 \text{ K})^4] \\
\left(\frac{1}{0.2} + \frac{1}{0.2} - 1\right) + N_{\text{shields}} \left(\frac{1}{0.15} + \frac{1}{0.15} - 1\right) \rightarrow N_{\text{shields}} = 2.92 \approx 3
\]

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A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at \( T_w = 500 \text{ K} \) shows a temperature reading of \( T_{th} = 850 \text{ K} \). Assuming the emissivity of the thermocouple junction to be \( \varepsilon + 0.6 \) and the convection heat transfer coefficient to be \( h = 560 \text{ W/m}^2 \cdot \text{C} \), determine the actual temperature of air.

Answer: 1111 K

Chapter 12, Solution 88

The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

**Assumptions** The surfaces are opaque, diffuse, and gray.

**Properties** The emissivity of thermocouple is given to be \( \varepsilon = 0.6 \).

**Analysis** The actual temperature of the air can be determined from

\[
T_f = T_{th} + \frac{\varepsilon h \sigma(T_{th}^4 - T_w^4)}{h} = 850 \text{ K} + \frac{(0.6)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(850 \text{ K})^4 - (500 \text{ K})^4]}{60 \text{ W/m}^2 \cdot \text{C}} = 1111 \text{ K}
\]