Estimation of insulating material’s thermophysical and radiational properties

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Thermophysical characteristics and emissivity of material are computed by solving the coefficient inverse problem of heat conduction using an iterative method. Data supplying is accomplished via temperature measuring by thermocouples on the analysing material. The type of boundary conditions and experimental information quantity should be satisfy the uniqueness of answer for estimating three temperature dependent thermophysical characteristics. Therefore it is needed transient heat flux and at least three temperature measurements in the specimen. The specimen is rectangular plate form with dimensions 40x40x15 mm. Because of construction quality and flat level it provides homogeneous, one-dimensional heat transfer. Boundary condition on one side is only temperature and on the other side simultaneously heat flux and radiation. The mathematical model is written as follows:

\[ C(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(T) \frac{\partial T}{\partial x} \right), \quad x \in (0,b) , \quad t \in (0,t_m) \]  
\[ T(x,0)=T_0 = \text{Const} \]  
\[ -\lambda(T(0,t)) \frac{\partial T(0,t)}{\partial x} = q_1(t) \]  
\[ -\lambda(T(b,t)) \frac{\partial T(b,t)}{\partial x} = q_2(t) - 4 \sigma \varepsilon(T(b,t)) T_4(b,t) \]  

where \( C(T) \) is the volumetric heat capacity as a function of \( T \) , \( \lambda(T) \) is the thermal conductivity, \( t \) is time (\( t_m \) final time) and \( \varepsilon(T) \) is emissivity.

Thermocouples measured temperatures are used to give additional information about its distribution in the specimen in order to solve inverse problem:

\[ T_{\text{meas}}(X_n,t) = \int_t^{t_m} f_n(t) \quad t \in (0,t_m) , \quad n=1,N \]  

The unknown functions are parametrized in cubic B-spline form:

\[ C(T) = \sum_{i=1}^{m_1} C_i \cdot \phi_{1,i}(T) \quad \lambda(T) = \sum_{i=1}^{m_2} \lambda_i \cdot \phi_{2,i}(T) \quad \varepsilon(T) = \sum_{i=1}^{m_3} \varepsilon_i \cdot \phi_{3,i}(T) \]  

where \( i \) is referred to number of homogeneous rectangular meshes constructed between \( T(\text{min}) \) & \( T(\text{max}) \) measured during experiment in the specimen.

So the next step is estimating a vector of parameters:

\[ \bar{P} = [C^1, C^2, ..., \lambda^1, \lambda^2, ..., \varepsilon^1, \varepsilon^2, ..., \varepsilon^m] \]

The least square residual function is :

\[ \bar{p} = \sum_{n=1}^{N} \int_{t_n}^{t_m} \left( T(X_n,t) - f_n(t) \right)^2 dt \]  

Inverse problem is ill-posed and the regularization property of optimization gradient methods are used. So the desired vector of parameters can be obtained from residual functional minimization (7) so that \( \| \bar{P} \| = \delta^2 \) where \( \delta^2 = \sum_{n=0}^{N} \int_{t_n}^{t_m} \sigma^2_n(t) \cdot dt \) is the integrated measurement error and \( \sigma_n \) measurement dispersion.