An inverse problem of identifying Robin's coefficient

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We consider in this paper the problem of determining Robin's coefficient $\varphi$, by using boundary measurements on a part $K$ of the boundary $\partial \Omega$, the results given in this work are of three kinds:

- Identifiability
- Stability
- Identification

Let $\Omega$ be a simply connected domain of $\mathbb{R}^2$ with a boundary $\partial \Omega$ a $C^{2,\beta}$ ($\beta > 0$) Jordan curve divided into three connected parts:

$$\partial \Omega = \tilde{\gamma} \cup \overline{\Gamma_D} \cup \overline{\Gamma_N}$$

Let be $K$ a subset of $\Gamma_N$ having an accumulation point $a$ which belongs to the interior of $\Gamma_N$.

The direct problem is therefore given by:

$$\begin{cases}
\Delta u = 0 & \text{in } \Omega \\
u = 0 & \text{on } \Gamma_D \\
\frac{\partial u}{\partial n} = \phi & \text{on } \Gamma_N \\
\frac{\partial u}{\partial n} + \varphi u = 0 & \text{on } \gamma
\end{cases} \quad (1)$$

where $\phi$ is a prescribed heat flux on $\Gamma_N$, $\psi \in H^{-\frac{1}{2}}(\Gamma_N)$; $\psi \not\equiv 0$ on $\Gamma_N$ and $\varphi$ is a continuous function on $\tilde{\gamma}$, to be determined by known $f = u \mid_K$.

In the first part of this work, we prove an optimal uniqueness result, improving the Holmgren's theorem for the Laplace operator. This result is therefore used to prove that the coefficient $\varphi$ can be determined by a single measure of the temperature $f$ on an infinity part $K$ of $\partial \Omega$.

In the following, we assume that $K$ is a non empty open subset of $\Gamma_N$.

The second section of this work is devoted to study the stability of this inverse problem: we prove a local Lipschitz stability of parameters $\varphi$ with respect to the boundary measurements.

Finally, we gives a numerical method to determine the coefficient $\varphi$: the inverse problem is turned into an optimization problem with respect to the parameter $\varphi$ by using a Kohn & Vogelius like cost function, the only minimum of which is proved to be the unknown coefficient $\varphi$. Furthermore, we prove that the derivative of this cost function with respect to a direction $\psi$ depends only on the state $u^0$, and not on its Lagrangian derivative $u^1(\psi)$. 

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