MONITORING THE WEAR-LINE OF A MELTING FURNACE

Karstein Sorli  
SINTEF Applied Mathematics  
N-7465 Trondheim NORWAY  
Karstein.Sorli@math.sintef.no

Inge M. Skaar  
NTNU Mathematical Sciences  
N-7491 Trondheim NORWAY  
skaar@math.ntnu.no

ABSTRACT

The paper includes a description of different strategies for monitoring the wear-line of a melting furnace. This type of industrial monitoring is important for both economic and safety reasons.

Temperature sensors at different locations in the lining of the furnace combined with an inverse heat conduction model are utilized for monitoring the state of the lining. The wear-line boundary is represented by a critical isotherm, and the estimation of this isotherm is the crucial part of the inverse solution. Two numerical algorithms for the inverse problem are described.

The algorithms are formulated and tested, both on model problem data and on measurement data acquired at an industrial furnace. One algorithm is based on utilizing a fixed boundary with controlling nodes, on the wearline side, to approximate the measurements. The other algorithmic approach is to approximate the wear-line as close as possible with as few curve representing parameters as possible. Both algorithms utilize the common approach of minimizing the sum of the squared residuals at the sensor locations.

We also report on some limited numerical sensitivity studies for the algorithms. The numerical experiments confirm the utility of the algorithms as the results are in good agreement with test model data as well as industrial data.

NOMENCLATURE

- $a_i$: parameters of the wearline definition
- $\Delta a_i$: edge lengths of hyper-tetrahedrons
- $c_i$: partial derivatives of $u_n$ w.r.t. $a_i$
- $h_a$: heat transfer coefficient between steel jacket and air [$W/m^2K$]
- $h_w$: heat transfer coefficient between steel jacket and water [$W/m^2K$]
- $H$: Hessian
- $J$: regulizer
- $k$: thermal conductivity [$W/mK$]
- $M$: $M+1$ is number of wearline definition parameters
- $N$: number of measurement locations
- $r$: spatial coordinate in radial direction [m]
- $u$: temperature [$^0C$]
- $u_a$: ambient temperature of air [$^0C$]
- $u_w$: ambient temperature of water [$^0C$]
- $u_n$: computed temperature in location $n$ [$^0C$]
- $U_n$: measured temperature in location $n$ [$^0C$]
- $v_i$: nodal values of temperature along control curve [$^0C$]
- $x$: radial component component of approximating cubic
- $y$: axial component of approximating cubic
- $z$: spatial coordinate in vertical direction [m]

Greek symbols

- $\alpha$: weighting factor of wearline curvature regularization
- $\alpha_0$: coefficient in regulizer for zero order term
- $\alpha_1$: coefficient in regulizer for first order term
- $\alpha_2$: coefficient in regulizer for second order term
- $\kappa$: curvature of wearline
- $\Gamma$: boundary of $\Omega$
- $\Gamma_i$: boundary segment $i$ of $\Omega$
- $\Omega$: region of computed heat conduction
- $\Phi$: sum of the squared residuals

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1 INTRODUCTION

The problem origins from an ilmenite melting furnace at Tinfos Titan & Iron KS in Tyssedal, Norway. The products from the melter are iron and titandioxide. Because the density of liquid iron is higher than that of liquid titandioxide, the slag (titandioxide) tap-holes are located above the iron tap-holes. The sidewall of the furnace is water-cooled and the bottom is air-cooled (see Figures 1 and 2). In time, the lining of the melter gets worn. This is due to chemical reactions and thermo-mechanical stress. It is crucial that one is able to monitor the wearing to avoid hot metal from breaking through the lining and cause damage to the melter and nearby equipment. Here we consider a rotational symmetric vertical section of the melter as illustrated in Figure 2. In one section there are 24 thermocouples installed to monitor the temperature development. The measurements are also used in a simple model to calculate the position of points on a specific isotherm. The purpose of this paper is to suggest methods to monitor the wearing of the ilmenite melter, i.e. to monitor the position of the wearline. Since we do not know exactly what defines the wearline, we instead try to locate a representative isotherm. Because the melting temperature of the iron is about $1450^\circ C$ and that of the titandioxide is about $1000^\circ C$, it is certain that the lining is not worn beyond the $1450^\circ C$ isotherm. Here we only consider stationary heat conduction. The transients are negligible in our monitoring timeframes. A system is built based on solving inverse steady heat conduction problems repeatedly for several sections of the furnace lining, with a frequency of 1 hour.

2 MATHEMATICAL MODEL OF HEAT CONDUCTION

The direct problem is to solve a well defined stationary heat conduction problem on a given domain $\Omega$ with boundary conditions, see Figure 3. In this specific case one can assume rotational symmetry, i.e.:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r k(u) \frac{\partial u}{\partial r} \right) + \frac{\partial}{\partial z} \left( k(u) \frac{\partial u}{\partial z} \right) = 0 \quad in \quad \Omega \quad (1)$$

where $u$ is the temperature at a point $(r, z) \in \Omega$, $r$ and $z$ being the radial and axial coordinates, respectively. The thermal conductivity, $k(u)$, is generally depending on the temperature. Equation (1) describes heat conduction in cylindrical coordinates when the angular direction component of the conduction is negligible. Even though the furnace lining heat conduction do not satisfy the assumption of axisymmetry in general, this assumption is locally valid and applicable along separate vertical sections of the lining, where a number of thermocouples are located. The boundary $\Gamma$ of $\Omega$ is split in 5 segments, as shown in Figure 3.

$$\Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5 = \Gamma \quad (2)$$

The boundary values for the problem are as follows. On $\Gamma_1$ the heat flux is zero, since the model is rotational symmetric,

$$\frac{\partial u}{\partial r} = 0 \quad on \quad \Gamma_1 \quad (3)$$

The bottom segment ($\Gamma_2$) is air-cooled so we have a mixed condition. The furnace is supplied with fans, blowing air underneath to increase the cooling of the bottom. $h_a$ is the heat transfer coefficient between the bottom and air, and $T_a$ is the ambient air
At the inside boundary (Γ₃) we have a Dirichlet condition. Because we are trying to monitor the wearline, we choose the function \( f(r,z) \) in (7) so as to describe the wearline. For instance, if the melting temperature of the metal inside the furnace is 1450°C we choose the function to be equal to this melting temperature.

In (Radmoser, 1998) a similar approach is used.

\[
- \kappa(u) \frac{\partial u}{\partial z} = h_w(u-u_{\text{w}}) \quad \text{on} \quad \Gamma_2
\]

The sidewall (Γ₂) is water-cooled so we have a mixed condition here as well. \( h_w \) is the heat transfer coefficient between the sidewall and water, and \( u_{\text{w}} \) is the ambient water temperature,

\[
- \kappa(u) \frac{\partial u}{\partial r} = h_w(u-u_{\text{w}}) \quad \text{on} \quad \Gamma_3
\]

At the upper end of the domain, the assumption of insulation is reasonable,

\[
\frac{\partial u}{\partial z} = 0 \quad \text{on} \quad \Gamma_4
\]

At the inside boundary (Γ₅) we have a Dirichlet condition. Because we are trying to monitor the wearline, we choose the function \( f(r,z) \) in (7) so as to describe the wearline. For instance, if the melting temperature of the metal inside the furnace is 1450°C we choose the function to be equal to this melting temperature.

In (Radmoser, 1998) a similar approach is used.

\[
\frac{\partial u}{\partial z} = 0 \quad \text{on} \quad \Gamma_5
\]

\[
u = f(r,z) \quad \text{on} \quad \Gamma_5
\]

3 ALGORITHMS FOR INVERSE HEAT CONDUCTION

The unknown wearline denoted by Γ₅, will be estimated utilizing a finite number of measurements

\[
U_n, \quad n = 1, \ldots, N
\]

at given locations in \( \Omega \), in combination with solving the mathematical model of heat conduction governed by Equations (1)-(7). The locations used in our studies are shown in Figure 3, i.e. a maximum of 9 sensor locations. The actual number of sensors in vertical sections of the industrial furnace is larger.

Many authors, e.g. (Beck et al., 1985), (Hensel, 1991), have analyzed inverse heat conduction problems, both analytically and numerically. In (Sorli & Olden, 1998) a method for finding heat transfer coefficients, based on a bi-section type algorithm for nonlinear and implicit equations, is presented. In (Radmoser, 1998) a similar problem to the present one is formulated and solved by a special type of angle constraint to regularize the problem.

In this section we present two different numerical algorithms for solving the given inverse problem. Both methods are based on the common approach of minimizing the sum of the squared residuals at the measurement points,

\[
\min_{n=1}^{N} \sum (u_n - U_n)^2
\]

This function is refined by regularization term(s) that are described in the following subsections.

3.1 Method of control boundary

The idea is that since we do not know exactly what the domain for the boundary-value problem looks like we just assume some fixed shape for the unknown boundary (Γ₅). We define this boundary as the control boundary. A rule is that the control boundary should be chosen such that it is as close to the desired isotherm as possible. This can be achieved by some a priori calculations or by starting with a very simple control boundary, e.g. a straight line and improve the guess from the results in an iterative manner. The inverse problem then reads: Find a temperature distribution along the control boundary which reconstructs the measured temperatures at the sensor positions. That is, given measurements \( U_n, n = 1, \ldots, N \), where \( N \) is the number of thermocouples, find a Dirichlet condition on \( \Gamma_5 \) which minimizes the sum of squares as given in (9). The values \( u_n, n = 1, \ldots, N \), are the calculated temperatures at the thermocouple locations for a given temperature distribution along \( \Gamma_5 \). Since inverse problems tend to be ill-posed it is not very likely that we will succeed in minimizing the square sum and get a reasonable solution to our problem. It is easily seen that the problem has similarities to the sideways heat equation which is ill-posed, that is, the solution does not depend continuously on the data, see (Beck et al., 1985) and (Hensel, 1991). To limit the behavior of the temperature distribution along \( \Gamma_5 \) we introduce a regularizer in addition to the square sum, see (Radmoser, 1998) and (Hensel, 1991). There-
fore, instead of minimizing the pure square sum we minimize

$$\Phi(v_1, v_2, \ldots, v_m) = \sum_{n=1}^{N} (u_{n} - U_{n})^2 + J$$  \hspace{1cm} (10)$$

with respect to the unknown boundary values $v_i, i = 1, \ldots, m$. The term $J$ is a regulator and $m$ is the number of nodes along the fixed control boundary. In calculations presented here $J$ is a finite difference approximation to

$$a_0 \int_{\Gamma_5} (u)^2 \, d\gamma + a_1 \int_{\Gamma_5} \left( \frac{\partial u}{\partial t} \right)^2 \, d\gamma + a_2 \int_{\Gamma_5} \left( \frac{\partial^2 u}{\partial T^2} \right)^2 \, d\gamma$$  \hspace{1cm} (11)$$

where $a_0, a_1$ and $a_2$ are constants. The constants are chosen large enough to stabilize the problem so iterations will converge. If it should happen that our guess for the control line is nearly large enough to stabilize the problem so iterations will converge. This will of course result in very fast convergence for the Newton-Raphson iterations, because we in fact are trying to solve a linear problem. The method of solving for unknown boundary values for linear systems are presented in (Hensel, 1991). The Newton-Raphson updates becomes

$$v_{i+1} = v_i - H^{-1}(v_i)F(v_i)$$  \hspace{1cm} (14)$$

and is symmetric. One advantage of the control boundary method is that the Jacobian becomes independent of the boundary values along $\Gamma_5$ when the thermal heat conductivity is independent of temperature. That is, we do only need to calculate the Jacobian once. This will of course result in very fast convergence for the Newton-Raphson iterations, because we in fact are trying to solve a linear problem. The method of solving for unknown boundary values for linear systems are presented in (Hensel, 1991). The Newton-Raphson updates becomes

$$v_{i+1} = v_i - H^{-1}(v_i)F(v_i)$$  \hspace{1cm} (14)$$

3.1.1 Test cases Two test cases, one with a “minor wearline” and one with a “severe wearline”, were created to test the algorithm. Table 1 shows the computed temperatures at the probe locations for both cases. Remember that wearline in this case is represented by the $1450 \degree C$ isotherm.

The upper plot of Figure 4 shows the calculated wearline, the actual wearline and the control line. In this case we tried to reconstruct a minor wearline using a linear control line rather far from the actual wearline, both in location and shape. The lower plot of Figure 4 shows the relative error at the locations of the thermocouples. The numbering of the thermocouples in Table 1 are as in Figure 3. The constants in the regularizer term were chosen to be $\alpha_0 = 0.0, \alpha_1 = 0.0005$ and $\alpha_2 = 0.0003$.

In Figure 5 the same wearline as in Figure 4 is reconstructed, but this time using a better fitted curved control line. Comparing the relative errors plotted in the lower plot of Figure 5 and 4 shows that a control line closer to the wearline, both with respect to shape and location, will contribute to a solution which fits the measured temperatures better. The same values for the regularization parameters as in the linear control curve case have been used to reconstruct the wearline in Figure 5.

For Case 2, i.e. “severe wearline”, the same tests have been performed. In Figure 6 a linear control line has been used in the reconstruction of the temperature distribution giving relative errors as in the lower plot of the Figure 6. Again, same values as before have been used for the regularization parameters.

We have also in the case of “severe wearline” tried to reconstruct the temperature distribution using a curved control line

$\begin{array}{cccc}
 n & r_n & z_n & U_{n} (1) & U_{n} (2) \\
 1 & 6.50 & 2.00 & 305.40 & 404.87 \\
 2 & 3.75 & 0.50 & 384.99 & 506.14 \\
 3 & 6.50 & 2.75 & 480.66 & 672.39 \\
 4 & 2.00 & 0.50 & 480.92 & 646.39 \\
 5 & 6.50 & 1.25 & 189.36 & 242.92 \\
 6 & 5.25 & 0.50 & 268.55 & 347.69 \\
 7 & 6.50 & 3.50 & 667.25 & 872.56 \\
 8 & 0.00 & 0.50 & 550.63 & 804.88 \\
 9 & 6.50 & 0.50 & 98.61 & 125.80 \\
\end{array}$

where $v_i = [v_1, v_2, \ldots, v_m]^T$. Table 1. SIMULATED TEMPERATURES AT NINE PROBE LOCATIONS FOR TEST CASE 1 ("MINOR" WEAR) AND TEST CASE 2 ("SEVERE" WEAR).
which is closer in location and shape to the critical isotherm. As was the case for the “minor wearline” this results in a better fit to measured temperatures, which is shown in the lower plot of Figure 7.

### 3.1.2 A brief response analysis

To analyze the response of temperature at measurement locations we introduced perturbations of the location of the boundary $\Gamma_5$. In this case the boundary condition for the inner boundary is $u = 1450^\circ C$. The conditions for the other boundaries are the same as for the test
cases. The inner boundary is described by quadratic polynomial

\[ z(r) = c_0 + c_1 r + c_2 r^2 \]  \hspace{1cm} (15)

The constants \( c_i, i = 0, 1, 2 \), are determined from the equations

\[ \frac{dz}{dr}(r_2) = 0, \quad z(r_1) = z_1 \quad \text{and} \quad z(r_2) = z_2 \]  \hspace{1cm} (16)

Then the temperatures at probe locations, \( n = 1, \ldots, 9 \), were simulated with \( r_1, z_1 = (0.0, 1.5) \) and \( r_2, z_2 = (6.0, 3.5) \). Now, to test how perturbations of the location of the isotherm (inner boundary) would effect the measured temperatures at probe locations, we simulated two cases. Case 1 with \( r_1, z_1 = (0.0, 1.5 + 0.025) \) and \( r_2, z_2 = (6.0 - 0.0025, 3.5) \), which is a curve just above the original. Case 2 with \( r_1, z_1 = (0.0, 1.5 - 0.025) \) and \( r_2, z_2 = (6.0 + 0.0025, 3.5) \), which is a curve just under the original curve.

In Figure 8 the two curves above and under the middle curve are given in the upper plot and corresponding relative errors (response) at probe locations in the lower plot. This shows that the relative errors in the test cases, at least for the curved control lines, are acceptable. That is, if one can tolerate deviations as in Figure 8.

### 3.2 Isotherm Cubic Curve Approximation

In this case the unknown location of an isotherm representing or approximating the wearline, is estimated in an iterative manner. First we formulate the algorithm with no other regularization than a parametric representation of this critical isotherm. This wearline parametrization is a special kind of regularization of the problem since we limit the number of parameters for describing the curve. Later we extend the regularization by including a curvature term to the minimization problem.

#### 3.2.1 Smooth wearlines

As suggested we apply the well-known concept of minimizing the sum of the squared residuals (see (9)) at the \( N \) measurement points. One approach of solving this problem could be to use the multidimensional down-hill simplex method (Nelder & Mead, 1965). This method only requires function evaluations, not derivatives. However, it is not very efficient in terms of the number of function evaluations it requires. Therefore, we have formulated another method based on the assumption of \( k \)-ca/l linear dependence

\[ u_n(a_1, \ldots, a_M) = a_0^n + a_1^n a_1 + \cdots + a_M^n a_M \]  \hspace{1cm} (17)

between the computed values \( u_n \) and \( M \) curve parameters \( a_1, \ldots, a_M \) defining the critical and unknown isothermal curve. In this work we have limited our study to cubic curves and present results for this case. However, extensions to cubic splines are proposed for wearlines that are changing quite rapidly on some parts.

In (17), for \( n = 1, \ldots, N \) the coefficients \( a_m^n, m = 0, \ldots, M \), are determined by multidimensional interpolation. \( M + 1 \) different sets of parameter values \( \{ a_j^0, \ldots, a_j^M \}, j = 0, \ldots, M \), give the following linear system of algebraic equations

\[
\begin{bmatrix}
1 & a_{j1}^0 & \cdots & a_{jM}^0 \\
1 & a_{j1}^1 & \cdots & a_{jM}^1 \\
\vdots & \vdots & \ddots & \vdots \\
1 & a_{j1}^M & \cdots & a_{jM}^M
\end{bmatrix}
\begin{bmatrix}
a_0^j \\
a_1^j \\
\vdots \\
a_M^j
\end{bmatrix} =
\begin{bmatrix}
t_0^j \\
t_1^j \\
\vdots \\
t_N^j
\end{bmatrix}
\]  \hspace{1cm} (18)

which may be solved by Gaussian elimination with backsubstitution or any triangular decomposition scheme. Observe that (18) must be solved for \( N \) different right-hand side vectors \( [t_0^N, \ldots, t_N^N]^T \), \( n = 1, \ldots, N \). However, the coefficient matrix is the same in each case. Therefore, the elimination only needs to be done once, while the backsubstitution is required for each of the different right-hand side vectors.

In the present study the parameter values \( \{ a_j^0, \ldots, a_j^M \}, j = 0, \ldots, M \), are given by

\[ a^0 = [a_0^0, \ldots, a_M^0]^T \]  \hspace{1cm} (19)

\[ a^j = a^0 + \Delta a_j s_j^T, \quad j = 1, \ldots, M \]  \hspace{1cm} (20)
forming a right-angled hyper-tetrahedron with \( M + 1 \) vertices in the \( M \)-dimensional Euclidean space. In (20) \( \Delta a_j \) is the edge length of the tetrahedron in the direction \( e_j = [0, \ldots, 0, 1, 0, \ldots, 0] \) where 1 is in the \( j^{th} \) position of the vector.

Let \( a^0 \) be our latest estimate of the curve parameters. We use an iterative formula to compute a new and improved estimate from the previous one. To solve the minimization problem of (9), the approach of computing the partial derivatives of this expression, with respect to \( \{a_j\}, j = 1, \ldots, M \) in this case, and setting the results equal to zero, is a common approach of finding the values minimizing a given function. In doing so we get

\[
f_m(a_1, \ldots, a_M) \equiv \sum_{n=1}^{N} (u_n - U_n) c_m^n = 0, \quad m = 1, \ldots, M
\]

(21)

where \( c_m^n = \frac{\partial g_m}{\partial a_m}, m = 1, \ldots, M, n = 1, \ldots, N, \) are given by (18). We solve this system iteratively by the well-known Newton-Raphson formula

\[
\begin{bmatrix} \frac{\partial f_1}{\partial a_1} & \frac{\partial f_1}{\partial a_2} & \cdots & \frac{\partial f_1}{\partial a_M} \\ \frac{\partial f_2}{\partial a_1} & \frac{\partial f_2}{\partial a_2} & \cdots & \frac{\partial f_2}{\partial a_M} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial a_1} & \frac{\partial f_M}{\partial a_2} & \cdots & \frac{\partial f_M}{\partial a_M} \end{bmatrix}
\begin{bmatrix} \Delta a_1^0 \\ \Delta a_2^0 \\ \vdots \\ \Delta a_M^0 \end{bmatrix}
= - \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_M \end{bmatrix}
\]

(22)

where both the right-hand side vector and the coefficient matrix are computed for \( a = a^0 \). The new iterate for the curve parameters is

\[
a^0 = a^0 + \Delta a^0
\]

where \( \Delta a^0 = [a_1^0, \ldots, a_M^0]^T \).

In order to reduce computing time we have split the iteration in two different cycles; one inner iteration cycle and one outer. During an inner iteration cycle we do not relocate \( a^j \), \( j = 1, \ldots, M \), but iterates only on \( \Delta a^j \). In the outer iteration, however, we do relocate the other parameter sets. The strategy we have chosen for ending an inner iteration cycle and doing an outer iteration step, is based on computing the distance from \( a^0 \) to the hyperplane spanned by \( a^j \), \( j = 1, \ldots, M \). If this distance is larger than some prescribed limit, and the number of inner iterations of the present cycle is below some limit, the inner iteration continues. When one of the limits are reached, the present inner iteration cycle stops, and an outer iteration is executed. The new \( a^j \), \( j = 1, \ldots, M \), parameter sets are still determined by (20), but now the \( \Delta a_j \) hyper-tetrahedron edge lengths are set to half the lengths of the previous ones. The latter choice may seem arbitrary, but it has proved to be quite efficient in the numerical tests we have done so far. However, it may prove more efficient to relate the reduction rate of the edge lengths of the hyper-tetrahedrons to the function values evaluated.

Any finite element or finite volume solver for direct heat conduction problems can be utilized for computing the \( u_n, n = 1, \ldots, N \), for given parameter sets \( \{a_1, \ldots, a_M\} \), i.e. for fixed \( \Gamma_5 \) (see Figure 3). In our validation tests a finite element solver and a triangular mesh generator were used. The main requirement of the solver is that it should handle curved boundaries sufficiently well. The mesh generator should create a quality mesh relatively fast since the mesh has to be regenerated several times for different regions. A typical mesh is shown in Figure 9. The mesh is refined towards \( \Gamma_5 \) due to the large temperature gradients close to this curve. The meshing is based on the Delaunay-Voronoi triangulation algorithm (see e.g. (Carey, 1997)).

3.2.2 Test cases - knowing the shape of the wearlines

Here we present results of using the algorithm presented above. The test regions are shown in Figure 10.

The cases (a) and (b) of Figure 10 are chosen so as to represent two extreme wear scenarios. They do not reflect real wear in existing industrial furnaces. Both these artificial wearlines are
created by a cubic curve of the form

$$x(s) = b_0 + b_1 s + b_2 s^2 + b_3 s^3, \quad 0 \leq s \leq 1$$

(23)

where

$$b_0 = x(0)$$

(24)

$$b_1 = x'(0)$$

(25)

$$b_2 = -3[x(0) - x(1)] - 2x'(0) - x'(1)$$

(26)

$$b_3 = 2[x(0) - x(1)] + x'(0) + x'(1)$$

(27)

This type of curve is often called Ferguson cubic curves (see e.g. (Davies & Samuels, 1996)). In the tests the following assumptions are made

$$x'(0) = (a_1, 0)$$

(28)

$$x'(1) = (0, a_2)$$

(29)

$$x(0) = (0, a_3)$$

(30)

$$x(1) = (a_4, H)$$

(31)

i.e., the slope of the wearline is horizontal at the left side and vertical at the right side of it. These are reasonable assumptions in real wear scenarios. In (31) $H$ is the height of the furnace wall. It is set equal to 3.5 in both tests. Also, in both cases $a_2 = 5$, $a_3 = 1.5$ and $a_4 = 6$. In case (a) $a_1 = 10$, while $a_1 = 26$ in the severe wear case (b).

The direct problem of (1) was solved for these two cases by a finite element solver. Numerical results were stored in positions 1 to 9 as depicted in Figure 3. These numbers were subsequently treated as measurements. This completes the preparation of the model tests.

The test of the given solution algorithm for the inverse problem is now concerned with estimating the wearlines of Figures 10a and 10b, using the same type of curve for estimating the wearline. However, now the parameters $a_1$ to $a_4$ are not known and will be estimated by the inverse algorithm. Hopefully, we are able to create the same parameter sets. Of course, this task is much easier than the real problem when the shape of the wearline is not known a priori. However, the test is still valuable, since it must succeed on this type of problem in order to handle arbitrary wearlines. The results of the tests are presented in Figure 11. Observe that the algorithm succeeds in the case of the non-smooth wearline in this case. It is slightly better for the severe wear case. However, for heavy wear it would probably be better to use cubic splines instead of a single cubic. This will be investigated in future.

The maximum relative error of the computed values at the measurement points is in this case less than 1% after only a couple of outer iterations (see description above). The initial estimates were chosen to enclose the exact parameter set with an initial hyper-tetrahedron having edge lengths of 10% of the absolute value of the exact parameters.

3.2.3 Modification for nonsmooth and real wearlines We now modify the algorithm with extra regularization as claimed at the beginning of this subsection. The reason for the modification is the need for handling general wearlines which are not smoothly shaped, i.e. with discontinuities in slope. For such cases the previous algorithm will easily fail to converge, due to sensitivity of the algorithm to “disturbances on the wearline”. This argument could also be turned around by saying that the algorithm is sensitive to disturbances in the measurements. The latter is obviously the case in real life, since every thermocouple-based measuring system produce data with some degree of noise. A rigorous analysis of this matter is not done, but is planned in future work.

We have formulated and tested a modification of the algo-
In the following tests we are not assuming any knowledge of the shape of the wearlines except for the following assumptions.

\[
\min \left( \sum_{n=1}^{N} (u_n - U_n)^2 + \alpha \int_{0}^{1} \kappa(s) ds \right)
\]  

(32)

Here \( \kappa \) is the curvature of the wearline

\[
\kappa(s) = (x_{ss} \cdot x_{ss})^{1/2}
\]

and \( \alpha \) is some weighting factor. Assuming a cubic wearline as in (23), its curvature is given by

\[
\kappa^2(s) = 2b_2 \cdot b_2 + 6b_2 \cdot b_4 \cdot s + 9b_3 \cdot b_3 s^2
\]

For the case of the 4-parametric problem formulated above the system of algebraic equations given in (21) is modified as

\[
\hat{f}_1 = \sum_{n=1}^{N} (t_n - U_n) c_1 + \alpha (4a_1 - 6a_4) = 0
\]

(33)

\[
\hat{f}_2 = \sum_{n=1}^{N} (t_n - U_n) c_2 + \alpha (4a_2 + 6a_3 - 21) = 0
\]

(34)

\[
\hat{f}_3 = \sum_{n=1}^{N} (t_n - U_n) c_3 + \alpha (6a_2 + 12a_3 - 42) = 0
\]

(35)

\[
\hat{f}_4 = \sum_{n=1}^{N} (t_n - U_n) c_4 + \alpha (12a_4 - 6a_1) = 0
\]

(36)

For the 4-parametric case the Newton iteration matrix of (22) is modified as

\[
\begin{bmatrix}
\frac{\partial \hat{f}_1}{\partial a_1} + 4\alpha & \frac{\partial \hat{f}_1}{\partial a_2} & \frac{\partial \hat{f}_1}{\partial a_3} & \frac{\partial \hat{f}_1}{\partial a_4} - 6\alpha \\
\frac{\partial \hat{f}_2}{\partial a_1} & \frac{\partial \hat{f}_2}{\partial a_2} + 4\alpha & \frac{\partial \hat{f}_2}{\partial a_3} & \frac{\partial \hat{f}_2}{\partial a_4} \\
\frac{\partial \hat{f}_3}{\partial a_1} & \frac{\partial \hat{f}_3}{\partial a_2} + 6\alpha & \frac{\partial \hat{f}_3}{\partial a_3} + 12\alpha & \frac{\partial \hat{f}_3}{\partial a_4} \\
\frac{\partial \hat{f}_4}{\partial a_1} - 6\alpha & \frac{\partial \hat{f}_4}{\partial a_2} & \frac{\partial \hat{f}_4}{\partial a_3} & \frac{\partial \hat{f}_4}{\partial a_4} + 12\alpha
\end{bmatrix}
\]

(37)

### 3.2.4 Test cases - not knowing the shape of the wearlines

In the following tests we are not assuming any knowledge of the shape of the wearlines except for the following assumptions

\[
y'(0) = \kappa'(1) = 0
\]

(38)

i.e., the wearline is horizontal at the left side and vertical at the right hand side, as in the previous case. The computed wearline is compared to the actual wearline in Figure 12.

Different values of \( \alpha \) were chosen and the respective history of iterations is shown in Figure 13. Numerical experiments indicate that \( \alpha \) should be larger than 0.5 in this case in order to get convergent results. For other wearlines and regions this value may change.

The maximum relative error of the computed values at the measurement points is in this case about 3%. The initial estimates were chosen to enclose the exact parameter set with an
initial hyper-tetrahedron having edge lengths of 10% the absolute value of the exact parameters.

4 RESULTS ON FURNACE DATA

In this section we present simulations of the wearline using actual temperature measurements from an ilmenite melting furnace. This is done with the method of control boundary. The method is tested on three different sets of measurements at different acquisition dates. There are some effects which can cause indifferences in comparison to measured temperatures other than the method itself. One effect is that the heat transfer coefficients are assumed to be $h_a = 30$ and $h_w = 150$ which is close to their actual values but not quite correct. Other contributions to error may be inaccuracy in location of thermocouples and air gaps in the brick-work. In Figure 14 the wearline is calculated using a linear control boundary. As expected the relative errors at the probe locations are rather large. But this calculation gives guidelines to how the position and shape of the control line should be. If the control line is placed closer to the actual wearline as in Figure 15 for the same set of measurements, we can observe some reduction in the relative errors. To get a better idea of how the actual wearline looks like, more effort should be made to get the control line closer to the wearline. This should be done because the relative errors are still too large. Figures 16 and 17 show the results of the control line method applied on another set of measurements. This time the curved control line in Figure 17 has a better fit than in Figure 15. This shows its effect on the relative errors at probe locations. We can observe a special effect in the upper part of the lining in Figure 17. This is caused by frozen titan-dioxide at the inside of the lining. This is a wanted effect because it protects the refractory against chemical attacks. Iron is not that easily frozen along the lining. One would have to increase the cooling of the sidewall significantly to get this effect. So one gets in a position where one would have to consider lining wear against heat loss. In Figures 18 and 19 the method of control line has been tested on yet another set of measured temperatures.

In the three cases presented in this section there is no reason to believe that any of the original refractory materials have been worn. The cases have been picked relatively early in the lifetime of the furnace lining. This lining has been operated for only a short period of time. The lifetime of such a lining should be somewhere between 15 and 20 years. At this stage there is no reason to believe that the lining should not reach its expected lifetime.

5 CONCLUSION

Numerical models for monitoring the wearline of the lining of a melting furnace have been developed. Two algorithms for solving the resulting inverse heat conduction problem are presented and tested, both on test problem data and on real measurement data sampled in an industrial furnace. Numerical experiments confirm the utility of both the control boundary algorithm and the algorithm of isothermal curve approximation. The reliability of the former one depends on the average distance between the curve and the actual wearline.
The tests of the control boundary method on minor and severe wearlines, show that if the control curve corresponds reasonably well to the actual wearline in location and shape, the relative errors become less than 2.5% in both cases. This is well within the acceptable bounds found in the response analysis. Since the control line can be relocated in an iterative manner to improve location and shape, this can be achieved in most cases.

The isotherm cubic curve approximation produces results comparable to results of the "close version" of the control boundary algorithm. The method requires limited knowledge of the actual wearline shape. Therefore, it is better suited to handle inverse problems with a complex behavior like rapidly changing
wearlines. The algorithm probably needs a modification for the most severe cases, and the utilization of splines are suggested. This will be investigated in future work.

Both algorithms need extra regularization terms when applied to real wearline problems. The control boundary algorithm adds regularization using common constraints on the temperature along the control boundary. The modified algorithm of isotherm cubic curve approximation includes a constraint on the curvature of the isotherm approximation. These modifications are shown to be both necessary and successful in numerical experiments.

The regularization term and the improving effects of locating the control line closer to the isotherm imply that the method of control boundary should be implemented as an iterative search for the isotherm. If that can be achieved, the regularization term vanish if the control line is exactly at the isotherm and we would only be minimizing the squared sum involving calculated and measured temperatures at probe locations.

The method of isotherm curve approximation as presented in this paper, is a promising stand-alone method for solving inverse heat conduction problems with a free boundary like the wearline in the present problem. An interesting and open question is whether this algorithm could be used as the method of iterative search for the optimal control boundary in the control boundary algorithm. Another possibility is to use the method of control boundary as a refinement method after using the method of isotherm approximation by cubics or splines. The potentiality of these ideas will be investigated in future work.

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