Turbulence Modeling

At a point in turbulent flow, the velocity components \((u, v, w)\) are fluctuating with time.

At any time, let

\[ u = U + u' \]

Assume that NS-5 (Newton-Stokes) eqns are valid, and are constant let \( U = U + u' \)

and \( V = V + v' \), etc., and take time average of NS-5 eqns:

\( \text{(incompressible)} \rightarrow \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + \frac{\partial}{\partial z} \left( u \frac{\partial u}{\partial z} + w \frac{\partial u}{\partial x} \right) = 0 \)

\[ \sum_j \frac{\partial}{\partial x_j} (u_j u') = R.H.S \]

NS terms are same in averaged form except for

\[ \sum_j \frac{\partial}{\partial x_j} (u_j u') \]

these terms give rise to "turbulent stress"
Idea: want to model
\[ \frac{\partial}{\partial x_j} \left( \overline{u_j u_i} \right) \]

in terms of \( U, V \) and their derivatives.

There are
1) Zero equation models
   a.k.a. algebraic models
2) One-equation models
3) Two-equation models

In reality solution of additional PDE in Domain
Some two-equation models
- \( K-E \) model
- \( K-W \) model
- \( K - turbulence \) kinetic energy
- \( \varepsilon - dissipation \)
- \( \omega - Vorticity \)

Mixing length model - simple idea due to Prandtl based on idea that turbulent flow moves in large "chunks" that mix together over some distance \( L \) in the mixing length.

Let turbulent shear stress be defined analogous to molecular shear stress
\[ \tau_{xy} = \mu_T \frac{\partial U}{\partial y} \]
Where the "eddy viscosity" $\nu_T$ is

$$\nu_T = \frac{1}{2} \frac{d \langle u \rangle}{dy}$$

so that

$$\mathbf{u} \cdot \mathbf{v} = \frac{1}{2} \frac{d \langle u \rangle}{dy}$$

The algebraic model gives a reasonable result for:

- confined flow (channel/pipes)
- jets
- wakes

Not good for recirculating flows

Recirculating flow

Fidap has several models:
1) mixing length
2) $k-\varepsilon$
3) $k$-$\omega$

Consult Tutorial Manual and Theory Manual for assistance
Solve pipe flow for turbulent case

Set

* TURBULENT in PROBLEM command
  - specify turbulent viscosity model
    - MIXING LENGTH
    - TWO-EQUATION, etc
  - specify "WALL" entity on walls

Look at FLUENT tutorial