Stability - 1

- Concepts

Equilibrium Positions

If you put a system in an equilibrium position, it will remain in the equilibrium position unless acted on by external inputs.

Mathematically: Equilibrium positions of \( \dot{x}(t) = f[x(t)] \) are

Equilibrium – Stable System

System has no friction – if system is “perturbed” away from equilibrium position, it will

“perturbed” positions

Stability of Equilibrium State

1. Stable

FIGURE 6.2 Stability of an equilibrium state.
Equilibrium – Asymptotically Stable System

Systems has friction – if system is “perturbed” away from equilibrium position, it will

Stability of Equilibrium State

2. Asymptotically stable

Equilibrium – Unstable System

Regardless of friction – if system is “perturbed” away from equilibrium position, it will
Multiple Equilibrium Positions

Nonlinear systems can have multiple equilibrium positions –

“perturbed”

positions

Multiple Equilibrium Positions

If you assume a point mass, $m$ on a massless rod of length $L$,

$\sum M_A = -mgL \cos \theta = J_A \ddot{\theta}$

Equilibrium Position #1

$\theta = +90^\circ$

Stability of Equilibrium State

4. Unstable

Figure 6.2 Stability of an equilibrium state.
Stability by Linearization #1a

\[ \ddot{\theta} + \frac{12g}{L} \cos \theta = 0 \]
Substitute  \( \theta = \theta_E + \delta \theta \)

\[ 0 + \frac{d^2}{dt^2} (\delta \theta) + \frac{12g}{L} (0 \cdot \cos \delta \theta - 1 \cdot \sin \delta \theta) = 0 \]

Stability by Linearization #1b

\[ \frac{d^2}{dt^2} (\delta \theta) - \frac{12g}{L} (\sin \delta \theta) = 0 \]

For small angles,

\[ \frac{d^2}{dt^2} (\delta \theta) - \frac{12g}{L} (\sin \delta \theta) = 0 \]

Take Laplace Transform and set ICs to 0,

Equilibrium Position #2

\[ \theta = -90^\circ \]

Stability by Linearization - #2a

\[ \ddot{\theta} + \frac{12g}{L} \cos \theta = 0 \]
Substitute  \( \theta = \theta_E + \delta \theta \)

\[ 0 + \frac{d^2}{dt^2} (\delta \theta) + \frac{12g}{L} (0 \cdot \cos \delta \theta + 1 \cdot \sin \delta \theta) = 0 \]
Stability by Linearization - #2b

\[ \frac{d^2}{dt^2}(\delta \theta) + \frac{12g}{L} \sin(\delta \theta) = 0 \]

For small angles, \( \sin(\delta \theta) \approx \delta \theta \)

\[ \frac{d^2}{dt^2}(\delta \theta) + \frac{12g}{L} \delta \theta = 0 \]

Take Laplace Transform and set ICs to 0,

Multiple Equilibrium Positions

Nonlinear systems can have multiple equilibrium positions.

Multiple Equilibrium Positions

**FIGURE 6.1** Equilibrium states.