Two Systems

\[ R(s) \overset{K}{\rightarrow} \frac{1}{s(x+6)} \overset{C(s)}{\rightarrow} \]

What is the difference between these two systems?

\[ R(s) \overset{K}{\rightarrow} \frac{1}{s(x+6)[0.001s+1]} \overset{C(s)}{\rightarrow} \]

Compare the Two Systems

- Simulate the systems with \( K=25 \)
  - is there any significant difference in the response?
- Plot the root locus for each system
  - where are the root loci essentially the same?
  - where are the root loci significantly different?
- Find the gain and phase margin for each system
  - what is the same?
  - where is different?

Compare Frequency Responses

\[
\begin{align*}
\text{w} &= \text{logspace(-2,4,100)}; \\
\text{num} &= [1]; \text{den1} = [1 6 0]; \\
\text{den2} &= \text{conv}([\text{den1}, [0.001 1]]) \\
\text{[mag1,phase1]} &= \text{bode}([\text{num}, \text{den1}, \text{w}]); \\
\text{[mag2,phase2]} &= \text{bode}([\text{num}, \text{den2}, \text{w}]); \\
\text{grid}('on') \\
\text{semilogx}(\text{w}, 20\times\log10(\text{mag1})), \text{w}, 20\times\log10(\text{mag2}) \\
\text{semilogx}(\text{w}, \text{phase1}), \text{w}, \text{phase2}
\end{align*}
\]

Problem #2 - SV Equations & TF

\[
\begin{align*}
\frac{d\Delta}{dt} &= \omega_m - \omega_L = \theta_m - \theta_L \\
\dot{\omega}_m &= \frac{1}{J_m} (\tau_m - K\Delta \theta) \\
\dot{\omega}_L &= \frac{1}{J_L} (-B_L \omega_L + K\Delta \theta) \\
\omega_L(s) &= \frac{K}{J_L J_m s^4 + B_L J_m s^2 + (J_L + J_m) K_s + B_L K}
\end{align*}
\]

Find Open-Loop Freq. Response

\[
\begin{align*}
\theta_m(s) &= \frac{J_L J_m s^4 + B_L J_m s^2 + (J_L + J_m) K s + B_L K}{s^3 J_L J_m + B_L J_m + (J_L + J_m) K s + B_L K} \\
J_L J_m &= 0.105 N m^2 s^{-2} \\
B_L J_m &= 1.275 N m^2 s^{-2} \\
(J_L + J_m) K &= 4250 N^2 m^{-2} s^{-2} \\
B_L K &= 42500 N^2 m^{-2} s^{-2}
\end{align*}
\]
Problem #2b - SV Eqns & TF

If we assume the shaft is “infinitely” stiff, $K \to \infty$, therefore $\Delta \theta \to 0$, but $K\Delta \theta$ is finite

$$\omega_m = \frac{1}{J_m} (t_m - K\Delta \theta)$$

$$\omega_L = \frac{1}{J_L} (-B_L \omega_L + K\Delta \theta)$$

Solve for $K\Delta \theta$ and substitute

$$\frac{\omega_m(s)}{\tau_m(s)} = \frac{\omega_L(s)}{\tau_L(s)} = \frac{1}{(J_L + J_m)s + B_L}$$

Both Frequency Responses

Find Open-Loop Freq. Response

$$\theta_d(s)$$

$$\frac{1}{(J_L + J_m)s + B_L}$$

$$\theta_L(s)$$

$$(J_L + J_m) = 0.85 N - m - s^2$$

$$B_L = 8.5 N - m - s$$

Conclusion

- “High” frequency dynamics can be ignored if we are interested in only “low” frequency responses
  - “high” and “low” are relative terms!
- **Always a part of system modeling**
  - no shafts/stiffnesses without some mass
  - no masses without some stiffness
  - no inductors without some resistance and capacitance
  - no capacitors without some resistance