Example #2

Closed-Loop Transfer Function is:

\[
\frac{C(s)}{R(s)} = \frac{K}{s(s + 20)} = \frac{K}{s(s + 20)+K}
\]

Use Matlab to plot Root Locus

Find K for roots with \(\zeta = 0.707\)

\[
|s + p_1| = |(-10 + j10) + 0| = \sqrt{200}
\]

\[
|s + p_2| = |(-10 + j10 + 20)| = \sqrt{200}
\]

Use Magnitude Criterion

\[
K \left( \frac{\text{Num}(s)}{\text{Den}(s)} \right) = \frac{1}{|s + p_1| \cdot |s + p_2|} = 1
\]

\[
K \left( \frac{1}{\sqrt{200} \cdot \sqrt{200}} \right) = 1 \Rightarrow K = 200
\]

Once a point, \(s\), is known to be on the root locus, then the magnitude criterion can be used to find the gain \(K\) for that point (i.e., find \(K\) at desired root locations)

Find K for a C.E. root at \(s = -8\)

\[
|s + p_1| = |-8 + 0| = 8
\]

\[
|s + p_2| = |-8 + 20| = 12
\]

\[
K \left( \frac{1}{8 \cdot 12} \right) = 1
\]

\[
\Rightarrow K = 96
\]

Example #3

Closed-Loop Transfer Function is:

\[
\frac{C(s)}{R(s)} = \frac{K(s + 3)}{(s + 1)(s + 2) + K(s + 3)}
\]

Use Matlab to find the root locus
Root Locus for Example #3

Modify Axes

Find K for minimum peak time

K for minimum peak time

Use magnitude criterion

Example #4

Closed-Loop Transfer Function is:

\[
\frac{C(s)}{R(s)} = \frac{K(s+5)}{(s+1)(s+3) + K(s+5)}
\]

Is the point \(s = -4 + j2\) on the root locus?
Use Phase Criterion

\[
\text{angle} \left( \frac{\text{Num}(s)}{\text{Den}(s)} \right) = 180^\circ \pm k360^\circ \\
\angle(s + z_1) - \angle(s + p_1) - \angle(s + p_2) = 180^\circ \pm k360^\circ \\
\angle(s + z_1) = \angle(-4 + j2 + 5) = \tan^{-1} \left( \frac{+2}{+1} \right) \\
\angle(s + z_1) = \tan^{-1} \left( \frac{+2}{+1} \right) = 63.4^\circ 
\]

Continue Phase Criterion

\[
\angle(s + p_1) = \angle(-4 + j2 + 1) = \tan^{-1} \left( \frac{+2}{-3} \right) \\
\angle(s + p_1) = \tan^{-1} \left( \frac{+2}{-3} \right) = 146.3^\circ \\
\angle(s + p_2) = \angle(-4 + j2 + 3) = \tan^{-1} \left( \frac{+2}{-1} \right) \\
\angle(s + p_1) = \tan^{-1} \left( \frac{+2}{-1} \right) = 116.6^\circ 
\]

Add Phase Angles

\[
\angle(s + z_1) - \angle(s + p_1) - \angle(s + p_2) = \\
63.4^\circ - 146.3^\circ - 116.6^\circ = -199.5^\circ \\
s = -4 + j2 \text{ is not on this root locus}
\]

Rules #1-#3 for Root Locus

- Put polynomial in standard form,
- Plot open-loop poles (X) and zeros (O) on complex plane
- Root locus is on the real axis to the left of an odd number of open-loop poles and zeros (gives angle of 180°)

Rules #4-#5 for Root Locus

- Number of linear asymptotes = n-m, angle to 1st one (equally spaced) is
  \[
  \phi_a = \frac{180^\circ}{n - m}
  \]
- If n-m>1, center of linear asymptotes found by
  \[
  \sigma_a = \frac{1}{n - m} \left( \sum_{i=1}^{n} (-p_i) - \sum_{j=1}^{m} (-z_j) \right)
  \]

Rules #6-#8 for Root Locus

- Determine the breakaway points on the real axis (if any).
- Use the Routh-Hurwitz criteria to evaluate imaginary axis crossings (if any)
  - look for a row of zeros in the \(S^0\) or \(S^1\) rows
- Evaluate the angle of departure from complex poles (and angle of arrival at complex zeros) by the angle criteria
Rules #9-#10 for Root Locus

- Sketch in the remainder of the root locus.
  - Use the angle criteria to determine suitability of any questionable points.
- The gain $K$ at any root location can be found by the magnitude criteria,
  \[
  K \frac{s + z_1}{s + p_1} \frac{s + z_2}{s + p_2} \cdots = 1
  \]

Example #5

closed-loop transfer function:
\[
\frac{C(s)}{R(s)} = \frac{K(s + 2)}{s(s + 1)(s + 3)(s + 4) + K(s + 2)}
\]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem

Example #6

closed-loop transfer function:
\[
\frac{C(s)}{R(s)} = \frac{1}{s(s + 4)(s + 25) + K}
\]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem

Example #7

closed-loop transfer function:
\[
\frac{C(s)}{R(s)} = \frac{K(s + 2)}{(s - 1)(s + 3)(s + 4) + K(s + 2)}
\]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem
Example #8

\[ R(s) + \frac{(s + 8)}{s(s + 1)(s + 2)} C(s) \]

Closed-Loop Transfer Function is:

\[ \frac{C(s)}{R(s)} = \frac{K(s + 8)}{s(s + 1)(s + 2) + K(s + 8)} \]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem.

Example #9

\[ R(s) + \frac{1}{s^2 + 4s + 8} C(s) \]

Closed-Loop Transfer Function is:

\[ \frac{C(s)}{R(s)} = \frac{K}{s^2 + 4s + 8 + K} \]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem.

Example #10

\[ R(s) + \frac{(s^2 + 2s + 101)}{s^2 + 2s + 17} C(s) \]

Closed-Loop Transfer Function is:

\[ \frac{C(s)}{R(s)} = \frac{K(s^2 + 2s + 101)}{(s + 5)(s^2 + 2s + 17) + K(s^2 + 2s + 101)} \]

Apply root locus steps 1, 2, 3, 4, 5, and 7 to this problem.