PID Loop Shaping Design Goals

- -20 dB/dec Slope in the Crossover Region
- High Gain at Low Frequencies
- Continued Decrease in Magnitude after the Crossover
  – we will discuss this issue at a later date
- Open-loop Crossover Equals (almost) Closed-Loop Bandwidth, $\omega_{pw}$

Slope in the Crossover Region

- If the slope is –20 dB/dec in the region near the crossover, then the open-loop transfer function phase should be around –90 deg.
  – Gives ~ 90 degree phase margin, good stability
- Schinstock’s rule of thumb for safe designs is
  – “–20 dB/dec for two decades; one before the crossover and one after.”

High Gain at Low Frequencies

- Allows output to closely follow (“track”) the desired input at low frequencies
- The higher the gain is the lower the error is, and therefore the tighter the closed-loop transfer function (CLTF) holds to a magnitude ratio of one (0 dB).

CLTF and Error TF

- $G(s) = \frac{1}{s} = OLTF$
- $T(s) = \frac{1}{1 + G(s)} = CLTF$
- $S(s) = \frac{1}{1 + G(s)} = ErrorTF$

$S(s) = \frac{1}{G(s)}$ for $G(s)$ large
$T(s) = G(s)$ for $G(s)$ small

Likely Design Steps

1. Choose desired bandwidth (crossover) frequency.
2. Choose “compensation” (the poles and zeros of the controller) to achieve the desirable shape.
3. Choose the “gain” to obtain the correct crossover.
**Voice Coil**

Voice coil is similar to a DC motor - linear force is proportional to the current.

\[ F = k_i j \]

**Simplified Model**

**System Transfer Functions**

**Open-Loop Transfer Function**

**Example Problem**

\[ m = 2 \text{ lbf-sec}^2/\text{in} \]
\[ b = 0.6 \text{ lbf-sec/in} \]

\[ k_d s^2 + k_p s + k_i \]

\[ k_d (s + z_1)(s + z_2) \]

**Design Goals**

- High low-frequency gain of the open-loop system (for good tracking)
- Closed-loop bandwidth \( \omega_{BW} \) of 50 Hz
- Phase margin of at least 70 degrees

\[ \text{PID} \Rightarrow k_d s^2 + k_p s + k_i = k_d (s + z_1)(s + z_2) \]
### Desired Open-Loop TF

**Desired Open-Loop TF #1**

- Set $z_2 = 31.4$ rad/sec
- Arbitrarily set $z_1 = 3$ rad/sec
- System pole ~ 0.3 rad/sec

**Desired Open-Loop TF #2**

- Set $z_2 = 31.4$ rad/sec
- Arbitrarily set $z_1 = 3$ rad/sec
- System pole ~ 0.3 rad/sec

### Finish the PID Controller

\[
R(s) \rightarrow k_d \frac{(s + 3)(s + 31.4)}{s} \rightarrow F \rightarrow \frac{1}{2s^2 + 0.6s} \rightarrow C(s)
\]

- Find the value of $k_d$ that gives the desired bandwidth of ~50 Hz
- What is the phase margin for this controller?

### Questions

- If the “force amplifier” saturated at 100 lb, what is the largest step input that will no cause saturation?
- Plot the response of the system to ±0.01 inch sine waves at
  - 1 Hz (6.28 rad/sec)
  - 5 Hz
  - 10 Hz
  - 25 Hz
  
  What is the max tracking error in each case?