Linearity

- Many real systems will exhibit linear (or nearly linear) behavior over some range of operation
- Linear system analysis is correct - at least over some portions of a system’s operating envelope
- Most “real” systems often operate outside of the linear region, and many of our common assumptions about system behavior no longer apply

Static and Coulomb Friction

- In linear system analysis, friction forces are assumed proportional to velocity, i.e. viscous friction
- If we have a motor velocity of zero, we should have no friction.
- In reality, a small amount of “static” (no velocity) or “Coulomb” friction is almost always present - even in roller or ball type “anti-friction” bearings
Static and Coulomb Friction

![Friction Force vs Velocity Graph]

Effects of Friction

- Some motor torque (or linear actuator force) “wasted” overcoming friction forces
  - leads to inefficiency from energy viewpoint
- As an actuator moves to its final location, the velocity approaches zero and the motor torque will balance frictional and gravity loads
- The actuator will come to slightly different final resting positions each time
  - depends on the final value of static friction
  - contributes to some loss of repeatability in motion
Eccentricity and Gear Errors

- Ideal relationships for gears assume that the point of gear contact remains at a fixed distance from the center of rotation.
- The true center of the gears “pitch” circle and the center of rotation will be separated by a small amount, known as the eccentricity.
- Small tooth-to-tooth errors also violate this assumption.

Eccentricity and Gear Errors

- The combination of these two effects can lead to a geometrical relationship between two gears like this.

\[ \theta_{\text{in}} \rightarrow \theta_{\text{out}} \]

--- actual

--- theoretical
Effects of Gear Errors

• If a position sensor is mounted on the actuator side, then the output is not “exactly” where the sensor measurement indicates
• Accuracy problems from this nonlinearity are minimized if the sensor is mounted on the output side of the gearing
• In extreme cases this can cause the control system to enter an oscillatory “hunting” phase

Gear Backlash

• Improper mounting (center-to-center distance) can give a small clearance between gear teeth
• When the input gear reverses direction, a small rotation is required before this clearance is removed and the output gear begins to move
• Gear backlash is just one of many phenomena that can be characterized as hysteresis
Gear Backlash - Hysteresis

- Clearance between shafts and bearings (at “pin” joints) can cause hysteresis.
- Hydraulic servovalves are also known for exhibiting this effect.

Saturation

- All actuators have some maximum output capability, regardless of the input.
- This reality violates the linearity assumption, since at some point we can “double” the input command, yet not “double” or even significantly change the output.
Industrial Controllers

- **PID controllers used in industrial settings often have different terminology:**

  \[ PID \rightarrow K_1 + \frac{K_2}{s} + K_3s \]

  - Proportional Band = \(100/K_1\)
  - Reset (=1/Integral) = \(K_2\)
  - Rate or Pre-act (Derivative) = \(K_3\)
Industrial Controllers - P.B.

- Proportional Band (P.B.) inherently recognizes the saturation of actuators:
  
  \[ \text{output} = K_1 \times \text{input} = \frac{100}{P.B.} \times \text{input} \]

- PID Controllers

- Each part of the PID contributes to the control effort differently
PID Controllers

• Proportional ($K_1$) contributes when $E(s)$ not zero

• Derivative ($K_3$) contributes when $E(s)$ is changing (during transients)
PID Controllers

• Integral ($K_2$) contributes when $E(s)$ has not been zero in the past

\[ R(s) + E(s) - C(s) \]

\[ \frac{K_1}{s} \]

\[ \frac{K_2}{s} \]

\[ K_3s \]

\[ \text{Actuator} \]

Integral Windup

• One other problem that can be caused by saturation is integral wind-up

\[ R(s) \]

\[ E(s) \]

\[ \frac{K_2}{s} \]

\[ C(s) \]

– if the actuator is saturated and the error is positive, $E(s) > C(s)$, the $K_2$ term will continue to integrate and build up a large output (“integral windup”)
Integral Windup

If the input $r(t)$ “reverses” and $e(t)$ becomes negative, it can take an excessively long time to integrate the negative error back to a level such that the actuator is no longer saturated.

Anti-Windup

“Clamp” the integral output at the actuator saturation level. A reversal in error can then be more quickly acted upon.