Types of Closed-Loop Control

• **Servo-mechanisms** require that the output \( C(s) \) “follow” the input \( R(s) \)
  - robot arms & machine tools
  - read/write “head” on hard disk storage device

• **Regulators** attempt to maintain a constant output in the presence of disturbances
  - automobile cruise control (& aircraft autopilot)
  - industrial process control (temperature, flow, pressure, etc.)

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**Servo-mechanisms**

\[ R(s) = \text{Input} = \text{“Desired” Output} \]
\[ C(s) = \text{Actual Output} \]

\[ r(t) \]
\[ c(t) \]

This line “implies” than an instantaneous measurement of the actual output \( C(s) \) is available to be subtracted from the “desired” input value \( R(s) \) --- Not always true!

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**Thickness Control**

Time delay, \( \tau_d = \frac{L}{v} \)

Control action applied here

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**Hot Water Heater with Remote Sense**

Figure 11.13 from *System Dynamics and Control*, E. Umez-Eronini, PWS Publishing
Hot Water Heater

System Equation (Conservation of Energy)
\[ C \frac{d\theta}{dt} = q - \frac{\theta}{R} \]
- \(C\) = thermal capacitance of water, \(\text{watt-sec}/^\circ C\)
- \(q\) = energy input to water from heater, \(\text{watt}\)
- \(R\) = thermal resistance of tank, \(^\circ C/\text{watt}\)
- \(\theta\) = relative temperature of tank, \((\theta = T_{\text{water}} - T_{\text{ambient}}), ^\circ C\)

Simulation #1
- **Simulate using Simulink™**

RC = 25 seconds (1st order time constant)

Simulation #2
- **Simulate using Simulink™**

Set \(\tau_d\) = 1 sec, 2 sec, 3 sec, 4 sec, 5 sec
Stable Unstable

Pure Time (transport) Delay, \(\tau_d\)

\[ G(s) = e^{-s\tau_d} \rightarrow \angle G(j\omega) = -\tau_d\omega \]

Stability Limits w/Delay

\[ \Phi_M = 0^\circ \]
\[ \Phi_M = 75^\circ \]
\[ \Phi_M = 50^\circ \]
Ziegler-Nichols PID Tuning

- The Ziegler-Nichols Closed-Loop Tuning Method looks at the response of the system under proportional only control to obtain “ideal” PID settings (leads to “quarter decay”).
  - Set up the system with proportional only control and add a disturbance (or change the input).
  - Alter the proportional gain process until you obtain the smallest gain which gives constant amplitude oscillations. This is the Ultimate Gain, \( K_{CU} \).
  - Find the period of these constant oscillations. This is known as the Ultimate Period, \( P_U \).

Ziegler-Nichols Gains

- Based on the “ultimate” gain, \( K_{CU} \), and the “ultimate” period, \( P_U \), set the PID (or just P or PI) controller gains as:

\[
P = K_c \left( 1 + \frac{1}{\tau_s} + \frac{1}{\tau_d} \right)
\]

<table>
<thead>
<tr>
<th>Type</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( K_c )</td>
</tr>
<tr>
<td>PI</td>
<td>( K_c + \frac{K_c}{\tau_s} )</td>
</tr>
<tr>
<td>PID</td>
<td>( K_c \left( 1 + \frac{1}{\tau_s} + \frac{1}{\tau_d} \right) )</td>
</tr>
</tbody>
</table>

\( K_c = K_{CU} \)

Sample Problem #1

- When \( \tau_d = 4.25 \) sec, \( K_{CU} = 10 \)
- What is \( P_U \)?
- What are Z-N gains \( (K_p, K_i, K_d) \)?

Sample Problem #2

- When \( \tau_d = 1 \) sec, what is \( K_{CU} \)?
- What is \( P_U \)?
- What are Z-N gains \( (K_p, K_i, K_d) \)?

Ideal and Approximate Derivative Terms

\[ G_{\text{ideal}}(s) = s \]

\[ G_{\text{approx}}(s) = \frac{1}{\tau s + 1} \]

Amplifies high frequency “noise” (which typically exists in real systems)

Low pass filter with break frequency \( \omega_b = 1/\tau \)