Types of Closed-Loop Control

- "Servo-mechanisms" require that the output $C(s)$ "follow" the input $R(s)$
  - robot arms & machine tools
  - read/write “head” on hard disk storage device

- "Regulators" attempt to maintain a constant output in the presence of disturbances
  - automobile cruise control (& aircraft autopilot)
  - industrial process control (temperature, flow, pressure, etc.)
Regulators

R(s) = Input ( = “Desired” Output)  N(s) = Disturbance

C(s) = Actual Output

N(s)

This line “implies” than an instantaneous measurement of the actual output C(s) is available to be subtracted from the “desired” input value R(s) --- Not always true!
Thickmess Control

This type of time delay also frequently called transport delay.

Velocity, $v$

$\text{Time delay, } \tau_d = \frac{L}{v}$

Control action applied here

Hot Water Heater w/Remote Sense

Power Supply & Temperature Controller

Velocity, $v$

Figure 11.13 from System Dynamics and Control, E. Umez-Eronini, PWS Publishing
Hot Water Heater

System Equation
(Conservation of Energy)
\[ C \frac{d\theta}{dt} = q - \frac{\theta}{R} \]

- \( C \) = thermal capacitance of water, watt-sec/°C
- \( q \) = energy input to water from heater, watt
- \( R \) = thermal resistance of tank, °C/watt
- \( \theta \) = relative temperature of tank, \((\theta = T_{\text{water}} - T_{\text{ambient}}), \degree C\)

Time Delay, \( \tau_d \)

\[ \theta_{\text{sensor}}(t) = \theta(t - \tau_d) \]

\[ \theta_{\text{sensor}}(s) = \theta(s)e^{-\tau_d s} \]
Simulation #1

- Simulate using Simulink™

RC = 25 seconds (1st order time constant)

Simulation #2

- Simulate using Simulink™

Set $\tau_d = 1$ sec, 2 sec, 3 sec, 4 sec, 5 sec

Stable       Unstable
Pure Time (transport) Delay, $\tau_d$

$$G(s) = e^{-\tau_d s} \rightarrow G(j\omega) = e^{-j\tau_d \omega} \Rightarrow |G(j\omega)| = 1 \quad \angle G(j\omega) = -\tau_d \omega$$

Stability Limits w/Delay

Stability Limit, $\Phi_M = 0^\circ$

$\Phi_M \approx 95^\circ$

$\Phi_M \approx 75^\circ$

$\Phi_M \approx 50^\circ$
Ziegler-Nichols PID Tuning

- The Ziegler-Nichols Closed-Loop Tuning Method looks at the response of the system under proportional only control to obtain “ideal” PID settings (leads to “quarter decay”).
  - Set up the system with proportional only control and add a disturbance (or change the input).
  - Alter the proportional gain process until you obtain the smallest gain which gives constant amplitude oscillations. This is the Ultimate Gain, $K_{CU}$.
  - Find the period of these constant oscillations. This is known as the Ultimate Period, $P_U$.

Ziegler-Nichols Gains

- Based on the “ultimate” gain, $K_{CU}$, and the “ultimate” period, $P_U$, set the PID (or just P or PI) controller gains as:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>PI</th>
<th>PID</th>
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<tbody>
<tr>
<td>$K_c$</td>
<td>$.5K_{cu}$</td>
<td>$.45K_{cu}$</td>
<td>$.6K_{cu}$</td>
</tr>
<tr>
<td>$\tau_I$</td>
<td>-</td>
<td>$P_u/1.2$</td>
<td>$P_u/2$</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>-</td>
<td>-</td>
<td>$P_u/8$</td>
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$$PID = K_c\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)$$

- Proportional term: $K_1 = K_P = K_{cu}$
- Integral term: $K_2 = K_I = \frac{K_{cu}}{\tau_i}$
- Proportional term: $K_3 = K_D = \tau_d K_{cu}$
Sample Problem #1

- When $\tau_d = 4.25$ sec, $K_{cu} = 10$
- What is $P_u$?
- What are Z-N gains ($K_p$, $K_i$, $K_d$)?

Set $\tau_d = 4.25$ sec for “continuous oscillations”

Sample Problem #2

- When $\tau_d = 1$ sec, what is $K_{cu}$?
- What is $P_u$?
- What are Z-N gains ($K_p$, $K_i$, $K_d$)?

Set $\tau_d = 1$ sec
Ideal and Approximate Derivative Terms

\[ \text{PID} = K_c \left( 1 + \frac{1}{\tau_s s + \tau_d s} \right) = K_p + \frac{K_i}{s} + K_d s \]

\[ G_{\text{ideal}}(s) = s \quad \text{Amplifies high frequency "noise" (which typically exists in real systems)} \]

\[ G_{\text{approx}}(s) = s \left( \frac{1}{\tau s + 1} \right) \quad \text{Low pass filter with break frequency } \omega_b = 1/\tau \]

Ideal and Approximate Derivatives (\( \tau = 1/100 \text{ sec} \))