We have already seen that a gain of $K=115$ (actually 123) gives dominant roots at $-2.0 \pm j3.7$

Is there a way to reduce the settling time while keeping the same peak time?

How can we force root locus to pass through here?
Add a zero to Controller

Open loop poles “push” the root locus to the left, away from the imaginary axis

Where do you to put $z_c$?

Phase or angle criterion forces root locus to a desired point
Compute known angles

Phase (angle) Criterion

Sum of angles from zeros = $\angle s+z_c$

Sum of angles from poles = $36.5^\circ + 90^\circ + 118.4^\circ$

$\angle s+z_c - 244.9^\circ = -180^\circ$

$\angle s+z_c = +64.9^\circ$

$\tan(64.9^\circ) = \left(\frac{3.7}{z_c - 4}\right)$

$z_c = 5.73$
Modified control system

Addition of the zero at $-z_c = -5.73$ forces the root locus to pass through the desired roots.

How do we find $K$?

$K$ is not 115!

Compute known angles
Use magnitude criterion

\[
K \left| \frac{\text{Num}(s)}{\text{Den}(s)} \right| = K \left| \frac{s + z_c}{s + p_1} \right| \left| \frac{s + p_2}{s + p_3} \right| = 1
\]

\[
K \frac{4.084}{6.22 \times 3.7 \times 4.206} = 1 \Rightarrow K \approx 23.7
\]

PD Controller

Add a derivative term to the controller
(makes a PD controller)

\[
K_3 = K, \quad K_I = Kz_c
\]
### PD Controller Simulation

Use Simulink\textsuperscript{TM} to simulate the response of the system to a unit step with $K_1 = 23.7 \times 5.73 = 136$, $K_f = 23.7$

### Problem 6, p. 515

Design a PD controller to reduce the settling time by a factor of 4 while continuing to operate the system with 20% overshoot.

*Find proportional $K$ that gives 20% overshoot first*
Lead Compensation

Add both an open loop pole and zero to improve transient performance

Note that $p_c > z_c$
Compute known angles

Arbitrarily place \(-z_c=-5\), compute location for \(-p_c\)

\[
s = -4 + j3.7
\]

Phase (angle) Criterion

Sum of angles from zeros = 74.9°

Sum of angles from poles = 36.5° + 90° + 118.4° + \angle s+p_c

74.9° - \angle s+p_c - 244.9° = -180°

\(-\angle s+p_c = -10°\)

\[
\tan(10°) = \frac{3.7}{p_c - 4}
\]

\(p_c = 25\)
Lead Compensator

\[ \frac{K \frac{s + 5}{s + 25}}{\frac{1}{(s + 2)(s + 4)(s + 9)}} \]

How do we find the appropriate value for \( K \)?

Compute known angles

\[ \sqrt{(3.7)^2 + (25 - 4)^2} \]
\[ \sqrt{(3.7)^2 + (9 - 4)^2} \]
\[ \sqrt{(3.7)^2 + (5 - 4)^2} \]
\[ \sqrt{(3.7)^2 + (2 - 4)^2} \]
Use magnitude criterion

\[
K \frac{|s + 5|}{|s + 25| * |s + 9| * |s + 4| * |s + 2|} = 1
\]

\[
K \frac{3.83}{21.32 * 6.22 * 3.7 * 4.21} = 1 \implies K \approx 540
\]

Lead Compensated System

Use \textit{Simulink}™ to simulate the response of the system to a unit step with \(K = 540\).
Question #1

Why bother with Lead Compensators when PD works better?

– “pure” differentiators not easy to physically implement (requires op-amp, see Table 9.10 on p. 501)
– the lead compensator can be built with passive components only (resistors and capacitors), and thus is easily implemented in analog control systems (see Table 9.11 on p. 503)

PID Controller Design

• Design process given on p. 476 of your text
• Three basic steps:
  – see if a simple P controller will work
  – if transient performance needs improvement, design a PD controller
  – if steady-state performance needs improvement, “cascade” a PI controller with the PD controller
Lead/Lag Controller Design

- **Design process given on p. 481-482 of text**
- **Three basic steps:**
  - see if a simple P controller will work
  - if transient performance needs improvement, design a lead compensator/controller
  - if steady-state performance needs improvement, “cascade” a lag compensator/controller with the lead compensator/controller