Controller Design via Freq. Response

- **Proportional control (K only)**
  - desired phase margin related to damping ratio
  - desired static error constants adjusted by K

- **Lag compensators**
  - reshape low frequency response to obtain desired error constant with required transient response

- **Lead compensators**
  - reshape “high” frequency response to get desired phase margin

lead compensation

\[
G_c(s) = \frac{1}{\beta} \frac{s}{\beta s + 1} \quad \beta < 1
\]

Definition - Bandwidth

- **Bandwidth** \(\omega_{BW}\), is the frequency at which the closed-loop frequency response is 3 dB below its “zero” frequency value (p. 579 of text)
  - “zero” frequency would be approximated by a very small frequency
  - if you roughly approximated your closed loop system as first order (i.e., a low pass filter), this would be the “break” frequency

Peak Time and Bandwidth

- **Lead Compensator Design** (p. 628)
  - Find closed-loop bandwidth \(\omega_{BW}\) required for transient performance
  - Set gain K of compensator to give desired steady-state error
  - Find \(\Phi_m\) for this gain K and determine how much phase required by lead comp.
  - Select \(\beta\) and T from requirements and re-draw the Bode plots
  - Reset overall gain due to change by lead
Problem #12 - initial

\[ C(s) \overset{+}{-} R(s) )20)(5( \]

When \( K = 10 \), the system has about 55% overshoot and a peak time of 0.5 sec. Find \( K \) for this initial setting.

Use Matlab™

Bode Plot with \( K = 1 \)

\[ R(s) \overset{K}{\rightarrow} C(s) \]

Bode Plot with \( K = 1000 \)

Closed-Loop Bandwidth

\[ R(s) \overset{1000}{\rightarrow} C(s) \]

Find the bandwidth (\( \omega_B \)) of the closed-loop system at this gain of \( K = 1000 \) Use Matlab™

Bandwidth ~10 rad/sec

Lead Compensation

10% OS \( \rightarrow \) \( \zeta = 0.59 \rightarrow \phi_M = 59^\circ \)

We have \( \Phi_M = 20^\circ \) need extra \( 39^\circ +11^\circ \)

\[ \omega_{\text{max}} = 50^\circ = \sin^{-1} \left( \frac{1-\beta}{1+\beta} \right) \]

\[ \beta = 0.13 \]

From \( K = 1000 \) plot, at -9 dB, \( \omega_{\text{max}} \approx 10.5 \) rad/sec

\[ \gamma_c(\omega_{\text{max}}) = 20 \log_{10} \left( \frac{1}{\sqrt{\beta}} \right) \]

\[ T = 0.26 \]

\[ \omega_{\text{max}} = \frac{1}{T \sqrt{\beta}} \]
**Lead Compensator**

\[ G_c(s) = \frac{1}{\beta} \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = \frac{1}{0.13} \frac{s + 0.26}{s + 0.13 \times 0.26} \]

\[ G_c(s) = \frac{7.7}{s + 29.3} \]

**Problem #12 - w/compensator**

\[ C(s) = \frac{1000 \cdot 7(s + 3.8)}{(s + 5)(s + 20)} \]

Check final answer with time domain (step input) solution

Use Matlab™

**Phase Margin w/Compensator #2**

\[ \Phi_M \approx 20° \]

From K=1000 plot, at -13 dB, \( \omega_{max} \approx 13 \) rad/sec

**Lead Compensation #2**

\[ G_c(s) = \frac{1}{\beta} \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = \frac{1}{0.05} \frac{s + 0.34}{s + 0.05 \times 0.34} \]

\[ G_c(s) = \frac{20}{s + 60} \]

**Problem #12 - w/compensator #2**

\[ C(s) = \frac{1000 \cdot 20(s + 3)}{(s + 5)(s + 20)} \]

Check final answer with time domain (step input) solution

Use Matlab™
Phase Margin w/Compensator

Bandwidth ~20 rad/sec

Phase Margin ~48°

Bandwidth ~20 rad/sec

-3 dB