Definitions - Unity Feedback

Open-loop transfer function

Magnitude criterion: $K \left| \frac{\text{Num}(s)}{\text{Den}(s)} \right| = 1 = 0\text{dB}$

Phase criterion: $\text{angle} \left( \frac{\text{Num}(s)}{\text{Den}(s)} \right) = -180^\circ$

Stability in Frequency Domain

- **Gain Margin, $G_M$** - How much open-loop gain $K$ can we add such that the 0 db crossover occurs at -180° phase?
- **Phase Margin, $\Phi_M$** - How much phase angle could we subtract at 0 dB crossover (unity gain) to reach -180° phase?

*Both gain and phase margin computed from the open-loop transfer function’s frequency response*
Estimations

- We can estimate some unity feedback, *closed-loop time responses* from the *open loop frequency response*:
  - damping ratio from phase margin
  - static error constants from low frequency slopes

Phase Margin and $\zeta$

- From p. 587 of text, for an ideal 2nd order system,

$$\Phi_M = \tan^{-1}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}\right)$$  \hspace{1cm} \text{Eqn. 10.73}

A plot of phase margin vs. damping ratio is shown in Fig. 10.48 and on next slide
Example #1

\[ \frac{K}{s(s^2 + 12s + 20)} \]

Find phase margin, \( \Phi_M \), for \( K = 30 \)

Use Matlab™

What are the closed loop roots at \( K = 30 \)?
Example #2

\[ R(s) \rightarrow \frac{K}{s(s^2 + 4s + 8)} \rightarrow C(s) \]

Find phase margin, \( \Phi_M \), for \( K = 10 \)

What are the closed loop roots at \( K = 10 \)?

\[ \left\{ \begin{array}{l}
\text{Use Matlab}^\text{TM}
\end{array} \right. \]

Static Error Constants

- Figure 10.51 of your text shows how to determine the static error constants from the low frequency response of the open-loop system
  - \( K_p \) (position error constant) for Type 0 systems
  - \( K_v \) (velocity error constant) for Type 1 systems
  - \( K_a \) (acceleration error constant) for Type 2 systems
Example #3

Type 0 system, so

\[ 20 \log_{10}(K_p) = 14 \text{dB} \]

\[ K_p = 10^{\frac{14}{20}} \approx 5 \]

Example #4

Type 1 system, so find \( K_v \)

Magnitude = 0 dB at \( \omega = 2 \), so \( K_v = 2 \) rad/sec
Definition - Bandwidth

- Bandwidth ($\omega_{BW}$), is the frequency at which the closed-loop frequency response is 3 dB below its “zero” frequency value (p. 579 of text)
  - “zero” frequency would be approximated by a very small frequency
  - if you roughly approximated your closed loop system as first order (i.e., a low pass filter), this would be the “break” frequency

Example #5

Find bandwidth, $\omega_{MW}$, for $K = 30$

Use Matlab™