Definitions - Unity Feedback

\[ R(s) + K \frac{\text{Num}(s)}{\text{Den}(s)} = C(s) \]

Magnitude criterion:

\[ K \left| \frac{\text{Num}(s)}{\text{Den}(s)} \right| = 1 = 0dB \]

Phase criterion:

\[ \text{angle} \left( \frac{\text{Num}(s)}{\text{Den}(s)} \right) = -180^\circ \]

Stability in Frequency Domain

- **Gain Margin, G_M** - How much open-loop gain K can we add such that the 0 db crossover occurs at -180° phase?
- **Phase Margin, \( \Phi_M \)** - How much phase angle could we subtract at 0 dB crossover (unity gain) to reach -180° phase?

Both gain and phase margin computed from the open-loop transfer function’s frequency response
Root Locus Example #7

Closed-Loop Transfer Function is:
\[
\frac{C(s)}{R(s)} = \frac{K}{s(s + 4)(s + 25)}
\]

Root locus shown on next slide

Limit for stability is \(K = 2900\)

Note that angles sum to 180° here!
Bode(num,den)

From: \( U(1) \) to: \( Y(1) \)

Gain Margin \(
\approx \) 70 dB

Phase Margin \(
\approx \) 90°

What is \( 20 \log_{10}(2900) \)?

-180° at 10 rad/sec

Gm=69.248 dB (at 10 rad/sec), Pm=89.834 deg. (at 0.01 rad/sec)
Example #9

Closed-Loop Transfer Function is:

\[
\frac{C(s)}{R(s)} = \frac{K}{s(s^2 + 4s + 8) + K}
\]

Root Locus #9

Note that angles sum to 180° here!

Limit for stability is \( K = 32 \)

Crosses at \( s = \pm 2.8 \text{ rad/sec} \)
Margin(num,den)

What is $20\log_{10}(32)$?

System Identification

- Use the experimentally determined frequency response to estimate the transfer function
  - apply sine wave inputs at known frequencies
  - measure the system output amplitude and phase angle
  - plot magnitude and phase as functions of frequency

\[
20\log_{10}\left(\frac{\text{output amplitude}}{\text{input amplitude}}\right)
\]
Step #1 in System ID

• **Low frequency slope**
  – if zero then no poles or zeros at origin
  – if +20 dB/decade, then a zero at origin
    • low frequency phase shift of +90°
  – if -20 dB/decade, then a pole at origin
    • low frequency phase shift of -90°

Step #2 in System ID

• **High frequency characteristics**
  – tells n-m (order of denominator - order of numerator)

\[
\text{high frequency slope} = -20(n - m) = 20(m - n)
\]
\[
\text{high frequency phase} = -90(n - m) = 90(m - n)
\]
Step #3 in System ID

- **Mid frequency characteristics**
  - look for breaks in either magnitude or phase
  - remember the characteristics of the remaining building blocks
    - 1st order pole
    - 1st order zero
    - 2nd order pole
    - 2nd order zero

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**Example #1**

![Plot of magnitude and phase vs. frequency](image_url)