Example 1

\[ Y(s) = \frac{1000}{(s + 4)(s + 500)} = \frac{1}{2} \cdot \frac{4}{s + 4} \cdot \frac{500}{s + 500} \]

- **Constant** = 1/2 \[ 20\log_{10}(1/2) = -6dB \]
- **1st order pole at s=-4**
- **1st order pole at s=-500**

**Linear Approximations**
Analytical Result

![Graph showing magnitude and phase angles vs frequency](image)

- \(|Y(j\omega)| = \angle \frac{1}{j\omega + 4} + \angle \frac{1}{j\omega + 500}

\[\angle Y(j\omega) = -\tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}\left(\frac{\omega}{500}\right)\]

- \(\angle Y(j\omega)\) goes from 0° to -180°
- \(\angle Y(j\omega)\) goes from 0° to -90°
Example #1 - Phase

![Phase Diagram](image)

Example 2

\[ Y(s) = \frac{1200(s + 9)}{s^2 + 50s + 10000} = \frac{1200 \times 9}{10000} \times \frac{s + 9}{9} \times \frac{10000}{s^2 + 50s + 10000} \]

- **Constant** = \(12 \times 9 / 100\) \(20 \log_{10}(12 \times 9 / 100) = +0.7\, dB\)

- **1st order zero at** \(s = -9\)

- **1st order pole at** \(s = -25 \pm j96.8\)
Linear Approximations

Example #2 - Magnitude
Phase Approx - 1st Order Zero

\[ \angle \frac{j\omega + 9}{1} \]
- First order zero

\[ \sim -90^\circ \text{ at } \omega = 90 \text{ rad/sec} \]
\[ \sim +45^\circ \text{ at } \omega = 9 \text{ rad/sec} \]
\[ \sim 0^\circ \text{ at } \omega = 0.9 \text{ rad/sec} \]

Phase Approx - 2nd Order Pole

\[ \angle \frac{1}{j\omega^2 + j50\omega + 10000} \]
- Second order pole

\[ \sim 0^\circ \text{ at } \omega = 10 \text{ rad/sec} \]
\[ \sim -90^\circ \text{ at } \omega = 100 \text{ rad/sec} \]
\[ \sim -180^\circ \text{ at } \omega = 1000 \text{ rad/sec} \]
**Example 3**

\[ Y(s) = \frac{1000(s + 40)}{(s + 10)(s + 80)(s + 200)} \]

\[ Y(s) = \frac{1 \times 4}{1 \times 8 \times 2} \frac{10}{(s + 10)} \frac{(s + 40)}{40} \frac{80}{(s + 80)} \frac{200}{(s + 200)} \]

- **Constant** = 3000
  \[ 20 \log_{10} (4/16) = -12.5 dB \]
- **1st order pole at** \( s = -10 \)
- **1st order zero at** \( s = -40 \)
- **1st order pole at** \( s = -80 \)
- **1st order pole at** \( s = -200 \)

**Linear Approximation - Ex #3**

![Linear Approximation Graph](image_url)
Example #3 - Magnitude

Magnitude, dB

Phase Approx- 1st Order Pole

\[ \frac{1}{j\omega + 80} \]

\( \sim 0^\circ \text{ at } \omega = 8 \text{ rad/sec} \)
\( -45^\circ \text{ at } \omega = 80 \text{ rad/sec} \)
\( -90^\circ \text{ at } \omega = 800 \text{ rad/sec} \)
Phase Approx- Example 3

Example #3 - Phase
Definitions - Unity Feedback

Magnitude criterion: \[ K \left| \frac{\text{Num}(s)}{\text{Den}(s)} \right| = 1 = 0dB \]

Phase criterion: \[ \text{angle} \left( \frac{\text{Num}(s)}{\text{Den}(s)} \right) = -180 ^\circ \]

Stability in Frequency Domain

- **Gain Margin, G_M** - How much open-loop gain K can we add such that the 0 dB crossover occurs at -180° phase?
- **Phase Margin, \( \Phi_M \)** - How much phase angle could we subtract at 0 dB crossover (unity gain) to reach -180° phase?

*Both gain and phase margin computed from the open-loop transfer function’s frequency response*
Root Locus Example #7

Closed-Loop Transfer Function is:

\[
\frac{C(s)}{R(s)} = \frac{K}{s(s + 4)(s + 25) + K}
\]

Root locus shown on next slide

Root Locus #7

Note that angles sum to 180° here!

Limit for stability is \(K=2900\)

Crosses at \(s=j10\) rad/sec
Bode(num,den)

**Bode Diagrams**

- Frequency (rad/sec)
- Phase (deg)
- Magnitude (dB)

From: U(1)  
To: Y(1)

**Gm=69.248 dB (at 10 rad/sec), Pm=89.834 deg. (at 0.01 rad/sec)**

-300  -250  -200  -150  -100  -50  0  50  100  150  200  250  300

-10  -1  10

What is $20\log_{10}(2900)$?

**Margin(num,den)**

Gain Margin ~ 70 dB

Phase Margin ~ 90°

-180° at 10 rad/sec
Example #9

Closed-Loop Transfer Function is:
\[
\frac{C(s)}{R(s)} = \frac{K}{s(s^2 + 4s + 8) + K}
\]

Root Locus #9

Note that angles sum to 180° here!

Limit for stability is \(K=32\)

Crosses at \(s=j2.8\) rad/sec
Gm = 30.103 dB (at 2.8284 rad/sec), Pm = 86.417 deg. (at 0.125 rad/sec)

Gain Margin ~ 70 dB

Phase Margin ~ 86°

-180° at 2.8 rad/sec

What is $20\log_{10}(32)$?