Frequency Response “Blocks”

• All transfer functions can be broken down to cascaded combinations of the following “building blocks”
  – constant
  – first order pole (or zero) at the origin
  – first order pole (or zero) not at the origin
  – second order pole (or zero) with damping ratio less than 1

Example

\[ Y(s) = \frac{750(s + 90)}{s(s^2 + 2s +101)(s + 500)} \]

• Constant = 750
• 1st order zero at s=-90
• 1st order pole at s=0
• 1st order pole at s=-500
• 2nd order pole at s=-1±j10
1st order pole at origin (s=0)

\[
G(s) = \frac{1}{s}
\]

\[
G(j\omega) = \frac{1}{j\omega}
\]

\[
|G(j\omega)| = \frac{1}{\omega}
\]

\[
\angle G(j\omega) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) = -90^\circ
\]

1st order pole

\[
G(s) = \frac{a}{s + a}
\]

\[
G(j\omega) = \frac{a}{j\omega + a}
\]

\[
|G(j\omega)| = \frac{a}{\sqrt{\omega^2 + a^2}}
\]

\[
\angle G(j\omega) = \tan^{-1}\left(\frac{0}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right)
\]

Note - graph drawn with \(a = 4\) rad/sec
2nd order pole

\[ G(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

\[ G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta \omega_n (j\omega) + \omega_n^2} \]

\[ |G(j\omega)| = \frac{\omega_n^2}{\sqrt{(2\zeta \omega_n \omega)^2 + (\omega_n^2 - \omega^2)^2}} \]

\[ \angle G(j\omega) = \tan^{-1}\left( \frac{0}{\omega_n^2} \right) - \tan^{-1}\left( \frac{2\zeta \omega_n \omega}{\omega_n^2 - \omega^2} \right) \]

Note - graph drawn with \( \zeta = 0.5, \omega_n = 4 \text{ rad/sec} \)

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1st order zero at origin

\[ G(s) = s \]

\[ G(j\omega) = j\omega \]

\[ |G(j\omega)| = \omega \]

\[ \angle G(j\omega) = \tan^{-1}\left( \frac{\omega}{0} \right) + 90^\circ \]
1st order zero

\[ G(s) = \frac{s + a}{a} \]

\[ G(j\omega) = \frac{j\omega + a}{a} \]

\[ |G(j\omega)| = \frac{\sqrt{\omega^2 + a^2}}{a} \]

\[ \angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{0}{a}\right) \]

Note - graph drawn with \( a = 4 \text{ rad/sec} \)

2nd order zero

\[ G(s) = \frac{s^2 + 2\zeta \omega_n s + \omega_n^2}{\omega_n^2} \]

\[ G(j\omega) = \frac{(j\omega)^2 + 2\zeta \omega_n (j\omega) + \omega_n^2}{\omega_n^2} \]

\[ |G(j\omega)| = \frac{\sqrt{(2\zeta \omega_n \omega)^2 + (\omega_n^2 - \omega^2)^2}}{\omega_n^2} \]

\[ \angle G(j\omega) = \tan^{-1}\left(\frac{2\zeta \omega_n \omega}{\omega_n^2 - \omega^2}\right) - \tan^{-1}\left(\frac{0}{\omega_n^2}\right) \]

Note - graph drawn with \( \zeta = 0.5, \omega_n = 4 \text{ rad/sec} \)
Linear Approximations

- Each of the “building blocks” can be approximated by one or two straight lines
  - for the 1st order poles and zeros at the origin, the “approximation” is exact
  - for the other cases the approximation is very good except where the two lines intersect
2nd order pole

Note - graph drawn with $\zeta = 0.2$, $\omega_n = 4$ rad/sec

Intersect at $\omega = \omega_n = 4$ rad/sec

1st order zero

Intersect at $\omega = \omega_i = 4$ rad/sec

+20 dB
2nd order zero

Intersect at \( \omega = \omega_z = 4 \text{ rad/sec} \)

\[ +40 \text{ dB} \]