Sections 4.1 – 4.3

Control Systems Engineering
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Poles and Zeros

Refers to transfer functions

• What is a zero?
  Value of s that makes the TF = 0

• What is a pole?
  Value of s that makes the TF → ∞

Example #1

• What are the zeros of the transfer function below?
  \[ G(s) = \frac{(s + 2)(s + 5)}{(s + 1)(s + 3)(s + 7)} \]

• What are the poles of the transfer function above?

Example #2

• What are the zeros of the transfer function below?
  \[ G(s) = \frac{(s^2 + 10s + 9)}{(s^3 + 11s^2 + 36s + 35)} \]

• What are the poles of the transfer function above?

Finding Roots of Polynomial

• Use roots command in Matlab:
  \[
  \text{EDU}> \text{roots( [ 1 11 36 35] )}
  \]
  \[
  \text{ans} =
  -5.9122
  -3.2865
  -1.8013
  \]
1st Order System Response

- **Typical 1st order systems**
  - 1 energy storing element
  - 1 element which dissipates (or removes) energy
- **a mass - damper combination**
- **a resistor - capacitor combination**
- **a resistor - inductor combination**

**Example #3**

Find the unit step response for the system below

\[
G(s) = \frac{C(s)}{R(s)} = \frac{s + 5}{(s + 7)}
\]

\[
C(s) = R(s) \frac{s + 5}{s(s + 7)} = \frac{s + 5}{s(s + 7)}
\]

1st Order Response

\[
C(s) = \frac{5/7}{s} + \frac{2/7}{s + 7} \rightarrow c(t) = \frac{5}{\tau} + \frac{2}{\tau} e^{-7t}
\]

1st Order System - Step Input

\[
\frac{dc(t)}{dt} + \frac{1}{\tau} c(t) = r(t) \rightarrow sC(s) - c_0 + \frac{1}{\tau} C(s) = \frac{A}{s}
\]

\[
\left( s + \frac{1}{\tau} \right) C(s) = \frac{A}{s} - c_0
\]

Let \( \frac{1}{\tau} = a \) \rightarrow \( C(s) = \frac{A-c_0}{s} + \frac{a}{s+a} \)

Example #4

- **Initial condition**, \( c_0 = 20 \)
- **Step input**, \( A = 60 \)
- **Time constant**, \( \tau = 1.25 \text{ sec}, \)
  \[ \rightarrow a = 1/1.25 \text{ sec}^{-1} \]

Find \( c(t) \)

\[
C(s) = \frac{A-c_0 s}{s(s+a)} \equiv \frac{?}{s} + \frac{?}{s+a}
\]
1st Order System - Step Response

- Response will be 63.2% closer to the final value after 1 time constant, $\tau$

1st Order System - Step Response

- Solution for a step (constant) input is given by

$$c = c_{ss} + \left[ c_0 - c_{ss} \right] e^{-t/\tau}$$

where
- $c_{ss}$ is the limiting or final (steady-state) value
- $c_0$ is the initial value at $t=0$
- $\tau$ is the time constant (units of seconds)

1st Order System - Step Response

- Response will be 63.2% closer to the final value after 1 time constant, $\tau$

1st Order System - Step Response

- $y_\infty = -60$ units
- $y_0 = 40$ units
- at $t = \tau$ (one time constant),

$$y = -60 + [40 - (-60)] e^{-1} = -23.2$$

Estimate the time constant from the graph

Example #5

- What happens when you have multiple 1st order poles?

$$G(s) = \frac{C(s)}{R(s)} = \frac{140}{(s+1)(s+7)(s+20)}$$

- Find solution, $c(t)$, for unit step input
Example #5

• Which of the three poles contribute “the most” to the output?

\[ G(s) = \frac{C(s)}{R(s)} = \frac{140}{(s+1)(s+7)(s+20)} \]

\[ \tau = 1 \text{ sec, “slow” pole} \]
\[ \tau = \frac{1}{7} \text{ sec, “faster” pole} \]
\[ \tau = \frac{1}{20} \text{ sec, “fastest” pole} \]

Simulink Simulation