Plotting and Graphing

Much of the data and information used by engineers is presented in the form of graphs. The values to be plotted can come from theoretical or empirical (observed) relationships, or from measured data. Properly presented, graphs provide a compact and concise delivery of information. However, poorly prepared graphs can be confusing and misleading. There are several computer packages available for producing graphs, but many of these do not generate adequate engineering style graphs without extensive modification to the default parameters. Remember that computers are simply tools, never use the excuse “that's what the computer gave me” for a poorly prepared graph.

The rules given below are for graphs with linear scales on both the horizontal and vertical axes. Rules for other scales are similar and will be presented in a later section.

Rules for Graphing

1) Graphs are almost always prepared by placing the dependent (output) value(s) along the ordinate (vertical axis) and the independent variable (input) along the abscissa (horizontal axis).

2) Scales for both axes should be selected for readability and clarity. Unless there are unusual circumstances, select scales that are multiples of 1, 2, 5, 10, 20, 50, etc. These scale values make the graph much easier to use and allow easier interpolation between data values.

3) It is usually desirable to show zero at the start of both the ordinate and abscissa unless this would compress the graph significantly. This rule is often difficult to apply, but use common sense ("engineering judgment") to determine if zero should be on both axes.

4) Graph paper should either be of the pre-printed type, carefully prepared blank paper, or computer-generated. If hand-drawn, axes should be dark and thick lines drawn with a straight edge for neatness and clarity. Markers should be placed regularly along each axis at the given scale divisions. If grid lines are used, they should be faint and unobtrusive.

5) Each axis should be indented from the edge of the graph paper to allow adequate lettering of the axis. Normally the abscissa is along the bottom of a graph and the ordinate along the left hand edge. If the graph requires a longer abscissa, the graph should be placed with the abscissa opposite the bound side of the graph and the ordinate along the left hand side.

6) Each axis of the graph must be labeled along with the units used. The axis lettering is usually placed outside the axis and its scale markers. Abscissa lettering is oriented normally and ordinate lettering is oriented such that it can be read if the paper is rotated 90 degrees clockwise.

7) Graphs of theoretical relationships (given by explicit formulas) do not have individual point designations, either solid, dashed, or dotted lines are used. Measured data are presented with individual symbols placed at each data point. Smooth curves or straight line segments are sometimes used to connect the data point symbols. Symbols can include filled and open circles, triangles, boxes, diamonds, etc., depending on how many different types of ordinate data are plotted. Empirical relationships are often plotted with symbols placed regularly along the abscissa, with smooth curves of the assumed type connecting the data points.
8) If more than one set of data is plotted, a legend is provided to identify each set of data. The legend is typically placed on the graph (away from any data points), below the abscissa axis, or to the right of the graph.

9) Most graphs should be titled, with the dependent variable name given first. For example, a graph entitled "Water Density vs. Temperature" would indicate that water density was the dependent (ordinate) variable and temperature was the independent (abscissa) value. The title should be placed on the graph where it will not interfere with the other information given. Note that in a report the figure title will be part of the caption, which is located immediately below the graph.

10) On a plot prepared by hand (i.e. not computer generated) the name of the graph preparer and the date are often placed in the lower right hand corner of the graph paper (not inside the graph axis boundaries).

**Log and Semi-log Plots**

In many sets of engineering or scientific data, input and/or output values are spread over an extremely wide range. For example, the data given on the right represents the idealized magnitude response of a simple low-pass filter. Plotting this data using a linear abscissa would compress the data greatly and make it difficult to see the rather sharp transition between 100 and 300 rad/sec. This type of data (with the abscissa values spread over several orders of magnitude) would normally be plotted on a semi-log graph. On a semi-log graph one of the axes (usually the abscissa) has a logarithmic scale. In engineering, this log scale is usually log base 10, although any other log scale is possible.

A quick review of logarithms is appropriate at this time. The definition of a logarithm can be developed from a simple equation:

\[ a^x = N \]

where \( a \) is any positive number except 1 and \( N \) is any positive number. Another equation can be defined from this first equation,

\[ x = \log_a (N) \]

which states that \( x \) is the logarithm base \( a \) of the number \( N \). A few simple examples using base 10 are:

\[ \log_{10}(10) = 1 \quad \log_{10}(100) = 2 \quad \log_{10}(1000) = 3 \]

since \( 10^1 = 10 \), \( 10^2 = 100 \), and \( 10^3 = 1000 \). Some of the commonly used properties of logarithms are given below:

<table>
<thead>
<tr>
<th>Natural Frequency (rad/sec)</th>
<th>Magnitude ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.9999</td>
</tr>
<tr>
<td>10</td>
<td>0.9988</td>
</tr>
<tr>
<td>30</td>
<td>0.9889</td>
</tr>
<tr>
<td>100</td>
<td>0.8944</td>
</tr>
<tr>
<td>300</td>
<td>0.5547</td>
</tr>
<tr>
<td>1000</td>
<td>0.1961</td>
</tr>
<tr>
<td>3000</td>
<td>0.0665</td>
</tr>
<tr>
<td>10000</td>
<td>0.0200</td>
</tr>
</tbody>
</table>
Many times the "natural" logarithm is used, and is commonly denoted by ln instead of log. Natural logarithms use the irrational number $e$ as a base, where $e$ is defined by

$$e = \lim_{x \to 0} (1 + x)^{1/x}$$

Logarithms to any base can be defined using the natural logarithm by the relationship

$$\log_a (N) = \frac{\ln(N)}{\ln(a)}$$

The original data table is given at the right with the addition of a column containing the log (log base 10) of the natural frequency. This set of data varies over a much smaller range of data values, and can be plotted on linear scales as shown in Figure 1 below.

<table>
<thead>
<tr>
<th>Natural Freq, $\omega$ (rad/sec)</th>
<th>log $\omega$</th>
<th>Magnitude ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.477</td>
<td>0.9999</td>
</tr>
<tr>
<td>10</td>
<td>1.000</td>
<td>0.9988</td>
</tr>
<tr>
<td>30</td>
<td>1.477</td>
<td>0.9889</td>
</tr>
<tr>
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<td>2.000</td>
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<tr>
<td>3000</td>
<td>3.477</td>
<td>0.0665</td>
</tr>
<tr>
<td>10000</td>
<td>4.000</td>
<td>0.0200</td>
</tr>
</tbody>
</table>

Figure 1. Magnitude ratio vs. log($\omega$).
A much simpler alternative to taking logarithms, then plotting on linear scales is to simply use logarithmic scales. Preprinted semi-log (1 log and 1 linear scale) and log-log (two log scales) graph paper is readily available in a variety of patterns. Many software packages (such as Excel) will generate plots directly with logarithmic axis on either the ordinate or abscissa (or both). The same data of magnitude ratio vs. frequency $\omega$ is plotted on logarithmic axes in Figure 2.

![Figure 2. Magnitude ratio vs. frequency, $\omega$ (log axis).](image)

To create a logarithmic axis in Excel, double click the axis, then check the logarithmic scale box within Format Axis. The minor gridlines must be turned on by checking the appropriate box in Chart Options.
Linearizing and Graphing Data

We have all used the familiar “straight line” as an approximation to a set of data. In your early chemistry and physics labs you also approximated data with other types of curves, such as exponential or logarithmic. Another class of problems can be solved using transformations to modify data such that the straight line approximation will work again.

A set of data (Speed vs. Time) is plotted below in Figure 3. There does not appear to be a good fit between the data points and any straight line.

![Figure 3. Speed vs. Time.](image)

However, if we look at the data carefully, we might note that a relatively simple transformation will allow us to plot a straight line. We form a column of $1/T$ as well as $T$ itself and plot Speed vs $(1/Time)$ in Figure 4. Note that a straight line approximation is much more reasonable with this transformed data.

![Figure 4. Speed vs. $(1/Time)$.](image)
A table of commonly encountered transformations is given below. Item (1) is simply the common straight line formula. The transformation used in the example above is given in item (3). Items (5) and (6) represent data that plots as a straight line on semi-log paper. Item (7) is represents data that plots as a straight line on log-log paper.

Table 1
Straight Line Transformations*

Note: \( Y = A + B X \)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>X</th>
<th>Intercept, A</th>
<th>Slope, B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>( y = a + bx )</td>
<td>y</td>
<td>x</td>
<td>a</td>
</tr>
<tr>
<td>2)</td>
<td>( y = a + b\sqrt{x} )</td>
<td>y</td>
<td>( \sqrt{x} )</td>
<td>a</td>
</tr>
<tr>
<td>3)</td>
<td>( y = a + b/x )</td>
<td>y</td>
<td>1/x</td>
<td>a</td>
</tr>
<tr>
<td>4)</td>
<td>( y = \frac{x}{a + bx} )</td>
<td>x/y</td>
<td>x</td>
<td>a</td>
</tr>
<tr>
<td>5)</td>
<td>( y = ab^x )</td>
<td>( \log(y) )</td>
<td>x</td>
<td>( \log(a) )</td>
</tr>
<tr>
<td>6)</td>
<td>( y = ac^{bx} )</td>
<td>( \log(y) )</td>
<td>x</td>
<td>( \log(a) )</td>
</tr>
<tr>
<td>7)</td>
<td>( y = ax^b )</td>
<td>( \log(y) )</td>
<td>( \log(x) )</td>
<td>( \log(a) )</td>
</tr>
<tr>
<td>8)</td>
<td>( y = a + bx^n )</td>
<td>y</td>
<td>( x^n )</td>
<td>a</td>
</tr>
</tbody>
</table>