Unit Conversion Examples

1. An \( M = 4.09 \) kg is attached to a spring with spring constant, \( K \). The natural frequency of the system is \( 2.38 \) Hz (or \( 2.38 \) cycles/sec). What is the spring constant, \( K \)?

\[
\omega_n = 2.38 \text{ cycles/sec} \left( \frac{2\pi \text{ rad}}{\text{cycle}} \right) = 14.96 \text{ rad/sec}
\]

\[
\omega_n = \sqrt{\frac{K}{M}} \rightarrow K = M\omega_n^2 = (4.09\text{ kg}) \left( 14.96 \text{ rad/sec} \right)^2 \left( \frac{1 \text{ N} \cdot \text{sec}^2}{1 \text{ kg} \cdot \text{m}} \right) \rightarrow K = 915 \frac{\text{N} \cdot \text{rad}^2}{\text{m}} = 915 \frac{\text{N}}{\text{m}}
\]

2. A mass that weighs \( 4.09 \) lb is attached to a spring with spring constant, \( K \). The natural frequency \( \omega_n = \sqrt{\frac{K}{M}} \) of the system is \( 2.38 \) Hz (or \( 2.38 \) cycles/sec). What is the spring constant, \( K \)?

\[
W = Mg \rightarrow M = \frac{W}{g} = \frac{4.09\text{ lb}}{32.2 \text{ ft/sec}^2}, \quad \omega_n = 2.38 \text{ cycles/sec} \left( \frac{2\pi \text{ rad}}{\text{cycle}} \right) = 14.96 \text{ rad/sec}
\]

\[
\omega_n = \sqrt{\frac{K}{M}} \rightarrow K = M\omega_n^2 = \left( \frac{4.09\text{ lb}}{32.2 \text{ ft/sec}^2} \right) \left( 14.96 \text{ rad/sec} \right)^2 \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) \rightarrow K = 2.37 \frac{\text{lb} \cdot \text{rad}^2}{\text{in}} = 2.37 \frac{\text{lb}}{\text{in}}
\]

3. A \( 60.0 \) lbm mass experiences a \( 9300 \) N force. Find the acceleration, \( a \) (\( f = ma \)) in units of \( \text{ft/sec}^2 \) and \( \text{m/sec}^2 \)

\[
a = \frac{f}{m} = \frac{9300 \text{ N}}{60.0 \text{ lbm}} \left( \frac{32.2 \text{ ft} \cdot \text{lbm}}{\text{sec}^2} \right) \left( \frac{1 \text{ lbf}}{4.448 \text{ N}} \right) = 1120 \frac{\text{ft}}{\text{sec}^2} \left( \frac{1 \text{ m}}{3.2808 \text{ ft}} \right) = 342 \frac{\text{m}}{\text{sec}^2}
\]

4. The kinetic energy of a point mass is given by \( \text{KE} = \frac{1}{2} m V^2 \), where \( m \) = mass, and \( V \) = velocity. Kinetic energy is typically expressed in US Engineering units of \( \text{ft-lbf} \), or the SI units of joules (J). With a \( 56 \) lbm mass and a velocity of \( 37 \) ft/sec,

\[
\text{KE} = \left( \frac{1}{2} \right) (56 \text{ lbm}) (37 \text{ ft})^2 \left( \frac{\text{lb} \cdot \text{s}^2}{32.178 \text{ ft-lbm}} \right) = 1190 \text{ ft} \cdot \text{lbf}
\]

\[
\text{KE} = 1190 \text{ ft} \cdot \text{lbf} \left( \frac{1 \text{ m}}{3.2808 \text{ ft}} \right) \left( \frac{4.448 \text{ N}}{1 \text{ lbf}} \right) = 1610 \text{ N} \cdot \text{m} \left( \frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}} \right) = 1610 \text{ J}
\]
5. The acceleration of gravity is 39 ft/s\(^2\) on Planet Zordar. Find the force of attraction (what we call weight, \(w = mg\)) of the gravitational field on a 25 lbm object in units of:

\[
\begin{align*}
w &= mg = (25\text{lbm}) \left(\frac{39}{s^2}\right) \left(\frac{\text{lbf} - \frac{s^2}{\text{lbm}}}{32.178\text{ft} - \text{lbm}}\right) \\
&= 30.3\text{lbf} \left(\frac{4.448\text{N}}{\text{lbf}}\right) = 135\text{N} 
\end{align*}
\]

6. The kinetic energy of a rotating mass is given by \(KE = \frac{1}{2} I \omega^2\), where \(I = \text{mass moment of inertia}\), and \(\omega = \text{angular velocity}\). With a mass moment of inertia of \(I = 6.50 \text{ lbm-ft}^2\) and an angular velocity of 1260 RPM,

\[
\omega = \left(\frac{1260}{\text{rev}}\right) \left(\frac{2\pi}{1\text{rev}}\right) \left(\frac{60\text{sec}}{1\text{min}}\right) = \frac{132}{\text{sec}} \text{ rad} 
\]

\[
KE = \left(\frac{1}{2}\right) \left(6.50\text{lbm} \cdot \text{ft}^2\right) \left(\frac{132}{\text{sec}}\right)^2 \left(\frac{\text{lbf} - \frac{s^2}{\text{lbm}}}{32.178\text{ft} - \text{lbm}}\right) = 1760 \text{ft} - \text{lbf} 
\]

7. The speed of sound in an ideal gas at temperature \(T\) is given by: \(c = \sqrt{(kRT)}\)

For air, \(k = 1.40\) (dimensionless) and \(R = 53.3 \text{ ft-lbf/lbm-°R}\). Find the speed of sound at \(T = 70^\circ\text{F} = 530^\circ\text{R}\) in units of ft/sec and mi/hr

\[
c = (kRT)^{0.5} = \left(1.4\right) \left(53.3\right) \left(\frac{\text{ft-lbf}}{\text{lbm-°R}}\right) \left(\frac{32.178\text{ft} - \text{lbm}}{\text{lbf} - \frac{s^2}{\text{lbm}}}\right)^{0.5} = 1130 \text{ ft/sec} 
\]

\[
c = 1130 \left(\frac{\text{ft}}{5280\text{ft}}\right) \left(\frac{1\text{min}}{60\text{sec}}\right) \left(\frac{1\text{hr}}{60\text{min}}\right) = 769 \text{ mi/hr} 
\]

8. The units for exhaust emissions from internal combustion engines are frequently given in \(\text{g/(hp-hr)}\). Convert the following to the equivalent in the SI system, \(\text{kg/(kW-day)}\)

\[
0.0376 \left(\frac{\text{g}}{\text{hp-hr}}\right) = 0.0376 \left(\frac{\text{g}}{\text{hp-hr}}\right) \left(\frac{1\text{kg}}{1000\text{g}}\right) \left(\frac{1\text{hp}}{0.746\text{kW}}\right) \left(\frac{24\text{hr}}{1\text{day}}\right) 
\]

\[
0.0376 \left(\frac{\text{g}}{\text{hp-hr}}\right) = 0.0376 \left(\frac{\text{g}}{\text{hp-hr}}\right) \left(\frac{1\text{kg}}{1000\text{g}}\right) \left(\frac{1\text{hp}}{0.746\text{kW}}\right) \left(\frac{24\text{hr}}{1\text{day}}\right) = 1.21 \times 10^{-3} \left(\frac{\text{kg}}{\text{kW-day}}\right) 
\]