Chapter 8
Strain Gages - Resistance

► The electrical resistance of most materials is a function of material properties and cross-sectional area.

\[
\frac{\Delta V}{\text{current}} \propto \frac{\text{cross-section}}{\text{resistance}}
\]

Strain Gages

► strain gages convert strain, change in length to original length = _______

► to change in resistance, change in resistance to original resistance = _______

Foil Element Strain Gage

► the ratio between strain and resistance is the gage factor,

\[
F = \frac{\Delta R}{R} = \frac{\Delta R}{\Delta L} \varepsilon \text{ or } \frac{\Delta R}{R} = \frac{\Delta L}{L}
\]

- for foil gages, \( F \approx 2 \)
- for semiconductor gages, \( F \approx 25 \) to 50

Strain Gage Mounting

Important Note:
► strain gages convert the strain seen ______ into a resistance change ______ by careful installation!

- usually mount gages by gluing (“SuperGlue”) or welding to structure

One Gage - Uniform Member in Pure Tension

► if we assume uniform loading across the width of the beam,

\[
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\]

One Gage - Pure Tension

► we have uniaxial stress, so

\[
\text{if we assume uniform loading across the width of the beam,}
\]

where \( E \) = modulus of elasticity,

- \( E_{\text{steel}} \approx 29 \times 10^6 \text{ lb/ft}^2 \approx 200 \times 10^6 \text{ N/m}^2 \)
- \( E_{\text{alum}} \approx 10.2 \times 10^6 \text{ lb/ft}^2 \approx 70 \times 10^6 \text{ N/m}^2 \)
One Gage - Pure Tension

► solving for strain,

\[ \varepsilon = \frac{P}{Ew} \cdot \frac{\Delta R}{R} = F \varepsilon \rightarrow \frac{\Delta R}{R} = \] 

- Find \( \Delta R \) if 
  - \( P = 3200 \text{ lb} \) (~ my pickup truck) 
  - \( F = 2.1, \ R = 120\Omega \) 
  - \( w = 1.0 \text{ inch}, \ t = 0.25 \text{ inch} \) 
  - material is steel

Wheatstone Bridges

Used to measure the small resistance changes created by strain gages

“Quarter” Bridge Circuit

1 of 4 “legs” of the Wheatstone Bridge is a ________________

“Quarter” Bridge Circuit

Left Side Voltage Divider

\[ V_i = \frac{R_A + R_B}{R_A} V_{in} \]
### Right Side Voltage Divider

\[ V_2 = \frac{R_{gage}}{R_C + R_{gage}} V_{in} \]

### Quarter Bridge Analysis

- normally make all fixed resistors equal,
  \[ R_C = R_B = R_A = R = \begin{cases} 120\Omega \\ 350\Omega \end{cases} \text{ or} \]
- gage output is “nominal” resistance + “delta” (change) resistance,
  \[ R_{gage} = \]

### Quarter Bridge Analysis

- substituting resistance values,
  \[ V_1 = \frac{R_B}{R_A + R_B} V_{in} = \frac{R}{R + (R)} V_{in} = \frac{1}{2} V_{in} \]
  \[ V_2 = \frac{R_C}{R_C + R_{gage}} V_{in} = \frac{R}{R + (R + \Delta R)} V_{in} \]
  \[ V_2 = \frac{1}{2 + \frac{\Delta R}{R}} V_{in} \]

### Quarter Bridge Analysis

- defining the output voltage,
  \[ V_{out} = V_1 - V_2 = \frac{1}{2} V_{in} - \frac{1}{2 + \frac{\Delta R}{R}} V_{in} \]
  \[ V_{out} = \frac{2 + \frac{\Delta R}{R}}{2 \left(2 + \frac{\Delta R}{R}\right)} V_{in} = \left(\frac{4}{4 + \frac{\Delta R}{R}}\right) V_{in} \]

### Quarter Bridge Analysis

- recall that the change in the gage resistance is very small,
  \[ \frac{2\Delta R}{R} \ll 4 \rightarrow \frac{2\Delta R}{R} \approx 4 \]
  \[ V_{out} = \frac{\left(\frac{\Delta R}{R}\right)}{4 + \frac{2\Delta R}{R}} V_{in} \approx \frac{\left(\frac{\Delta R}{R}\right)}{4} V_{in} \]
  \[ “quarter” \text{ bridge} \]

### Two Gages - Cantilever Beam

- If mounted correctly, the 2 gages “see” the same strain magnitude, where
  - one gage in tension (R _________)
  - one gage in compression (R _________)
“Half” Bridge Circuit

2 of 4 “legs” of the Wheatstone Bridge are strain gages

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Half Bridge Analysis

► substituting resistance values,

\[
V_1 = \frac{R}{R+2\Delta R} V_{in}, \quad V_2 = \frac{R-\Delta R}{(R+\Delta R)+(R-\Delta R)} V_{in}
\]

\[
V_{out} = V_1 - V_2 = \left(\frac{R}{2R} - \frac{R-\Delta R}{2R}\right) V_{in}
\]

\[
V_{out} = \frac{\Delta R}{R} V_{in}
\]

Four Gages - Cantilever Beam

► two gages (on top) in tension (R ______) and
► two gages (bottom) in compression (R ______)

“Full” Bridge Circuit - Pure Bending

► gages of opposite strain in adjacent legs of bridge,

\[
V_{out} \approx \frac{\Delta R}{R} V_{in}
\]

Find \( \sigma, \varepsilon, \Delta R, \) and \( V_{out} \) if

- \( P = 50 \text{ lb}, \ F = 2.0, \ R = 350\Omega \)
- \( w = 1.0 \text{ inch}, \ t = 0.25 \text{ inch}, \ x = 6.0 \text{ inch} \)
- material is aluminum (\( E_{\text{Al}} \approx 10.5 \times 10^6 \text{ psi} \))
- \( V_{in} = 12.1 \text{ volts} \)

“Full” Bridge Circuit - Pure Bending

All 4 “legs” of the Wheatstone Bridge are strain gages

Biaxial Stress/Strain

Thin-walled “pressure vessel” equations:

\[
\sigma_{hoop} = \sigma_L = \frac{P t}{2 \pi R h} = \frac{E \varepsilon_{hoop}}{\nu + \varepsilon_L}
\]

Biaxial Strain Equations:

\[
\varepsilon_L = \frac{\sigma_L}{E} = \frac{nu \varepsilon_{hoop}}{E}
\]

\[
\varepsilon_{hoop} = \frac{\sigma_{hoop}}{E} = \frac{\varepsilon_L}{\nu}
\]