Homework Problem 5-4 b

Steel Plate

\[ h \text{ Tm} \quad h \text{ Tm} \]

\[ -L \quad +L \]

Cooled on both sides so given thickness = \( \frac{2L}{\text{Thick = 30mm so } L = 15 \text{ mm } \approx 0.015 \text{ m}} \)

Check for Lumped capacitance

\[ \text{Bic}_{LC} = \frac{h_{AVG}}{k} = \frac{hL}{k} \]

\[ \text{Bi} = \frac{\text{Thick} \times L}{k} = \frac{0.015 \times 150}{16.3} = 0.138 \]

L.C. may give reasonable answer BUT lets do spatially dependent soln as example

\[ F_0 = \frac{x^2}{L^2} > 0.2 \]

Then \[ \frac{\theta}{\theta_0} = C_1 \exp(-\xi_1 F_0) \cos \left( \xi_1 \frac{x}{L} \right) \]

\[ C_1, \xi_1 \text{ depend only on } \text{Bic}_L \]

\[ \xi_1 = 0.3619 \quad C_1 = 1.0219 \]
\[ F_{\text{reqd}} = \ln \left( \frac{\theta / \theta_0}{c_1} \right) \times \frac{1}{S_i} \]

\[ = \ln \left( \frac{50 - 20}{500 - 40} \right) \times \frac{1}{(3619)^2} \]

\[ \frac{\alpha L^2}{\lambda} = F_{\text{reqd}} = 13.5 > 0.2 \quad \text{(Solu l-ferm)} \]

\[ \alpha = 4.4 \times 10^{-6} \text{ m}^2/\text{s} \]

\[ t = \frac{F_0 \times L^2}{\alpha} = \frac{13.5 \times (0.015)^2 \text{ m}^2}{4.4 \times 10^{-6} \text{ m}^2/\text{s}} = 690 \text{ s} \]

\[ \approx 11.5 \text{ min} \]

b) What is surface temp at \( t = 690 \text{ s} \)?

\[ \frac{\Theta}{\Theta_0} = C_1 \exp \left( -F_0 \times \frac{\Theta^2}{\Theta_0} \right) \times \cos \left( \frac{\Theta}{L} \right) \]

Note:

\[ \frac{\Theta_L}{\Theta_{\text{center}}} = \frac{\cos (\Theta_{\text{center}})}{1} = 0.999998 \approx 1.0 \]

C) \( \text{Center} \approx \text{Edge} \) so \( t \approx 11.5 \text{ min} \)

\[ t = \frac{8CV}{kA} \]

\[ L_c = \frac{A}{\pi D^2/4} \times \frac{(\pi D^2/4)L}{\pi D L + 2 \times \pi D^2/4} \]

\[ D = \frac{L}{2} \]

\[ L_c = \frac{D}{4(1 + \frac{D}{2L})} = \frac{L}{8(1 + \frac{1}{4})} \]
\[ L_c = \frac{4}{110} \]

\[ z = \frac{1000 \beta}{J/\text{cm}^2} \]

\[ \frac{\theta}{\theta_0} = \exp\left(-\frac{t}{T}\right) = \]

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Turns out that 1-D solutions can be combined to analyze 2-D or 3-D situations.

\[ \Theta = \Theta(\theta, r, t) = C(r(t)) \times P(x(t)) \]

\[ \frac{h \cdot D^2}{4} \times L = \frac{\pi \cdot D^3}{2} \]

\[ D = \frac{3 \times 350E-6 \times 2}{\pi} \]