Derived for a fin w/o heat generation:

\[ \frac{d^2T}{dx^2} = \frac{hP}{kAc} (T - T_{env}) \]

So if we include \( q^{"} \) then

\[ \frac{d^2T}{dx^2} + \frac{q^{"}}{k} = \frac{hP}{kAc} (T - T_{env}) \]

B.C.'s:

\( x=0 \) \( T=T_0 \) or \( T(0)=T_0 \)

\( x=L \) \( q_{conv} = q_{conv} \)

Fourier Law

\[ -k \frac{dT}{dx} \bigg|_{x=L} = \hat{q}_{conv} \]

How to solve?

Let \( \Theta(x) = T(x) - T_{env} \)

Then

\[ \frac{d^2\Theta}{dx^2} + \frac{q^{"}}{k} = \frac{hP}{kAc} \Theta \]

Look for \( \Theta(x) = \Theta_h(x) + \Theta_p(x) \)
\[ \frac{d^2 \Theta_n}{dx^2} = \frac{k}{kAcs} \Theta_n \]

Solution is:
\[ \Theta_n(x) = C_1 e^{-mx} + C_2 e^{+mx} \]

\[ m = \sqrt{\frac{hp}{kAcs}} \]

\[ \Theta_p(x) = ? \]

\[ \Theta_p(x) = A x + B \]

\[ \frac{d \Theta_p}{dx} = A \]

\[ \frac{d^2 \Theta_p}{dx^2} = 0 \]

Assign this form to substitute into equation:
\[ 0 + \frac{q_{\text{ext}}}{k} = \frac{hp}{kAcs} (Ax + B) \]

\[ 0 \cdot x' + \frac{q_{\text{ext}}'}{k} = (A) \frac{hp}{kAcs} x + B \frac{hp}{kAcs} \]

Equate like terms on both sides:
\[ 0 = A \left( \frac{hp}{kAcs} \right) \]

\[ \Rightarrow A = 0 \]

So:
\[ \Theta_p(x) = \frac{q_{\text{ext}}}{hp} Acs \]

And:
\[ \Theta(x) = \Theta_n(x) + \Theta_p(x) \]

\[ = C_1 e^{-mx} + C_2 e^{+mx} + \frac{q_{\text{ext}}}{hp} Acs \]

Use B.C.s to find \( C_1, C_2 \)
\[ \Theta(0) = T_0 - T_{aw} = \Theta_0 = C_1 + C_2 + \frac{q_0}{hp} \]

relating \( b/w \) \( C_1 \) and \( C_2 \)

other B.C.

\[ k \left( \frac{1}{L} + \frac{mC e^{-ml} + C_2 e^{ml} + q_0 Acs}{hp} \right) = \]

\[ \frac{d^2 \Theta}{dx^2} \]

\[ h \left( C e^{-ml} + C_2 e^{ml} + q_0 Acs \right) \]

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Pick up discussion on extended surface (w/o heat generation)

\[ \frac{d^2 \Theta}{dx^2} = m^2 \Theta \]

\[ m^2 = \frac{hp}{kAcs} \]

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Start solution is

1. \( \Theta(x) = C_1 e^{-mx} + C_2 e^{mx} \)

Recall \( \cosh(x) = \frac{e^x + e^{-x}}{2} \)

\( \sinh(x) = \frac{e^x - e^{-x}}{2} \)

Alternate form of \( \cosh(x) \)

2. \( \Theta(x) = C_3 \cosh(x) + C_4 \sinh(x) \)
How to find constants?
Use Boundary conditions.
One B.C. always will be \( T(0) = T_0 = T_0 \)
So \( \theta(0) = T_0 - T_{\text{env}} = \theta_0 \)
What's going on at \( x = L \) ???
Four cases to consider

(A) Fin is "very long"
   as \( x \to \infty \)
   \( \theta \to 0 \)
   For form of (1), see at \( x \to \infty, \theta \to \infty \)
   if \( C_2 \neq 0 \)
   So, for a long fin, must have \( C_2 = 0 \)
   \( \theta(x) = C_1 e^{-mx} \)
   use \( \theta(0) = \theta_0 = C_1 e^{-m0} = C_1 \)
   \( \theta(x) = \theta_0 e^{-mx} \) "Long" fin

(B)