Least time: introduced the \( T-ds \) relations

\[
Tds = du + Pdv \quad (1) \\
Tds = dh - vdp \quad (2)
\]

Thermodynamic identities

Consider an ideal gas

\[
Pv = RT \quad \text{eqn of state}
\]

\[
\Delta w = \int c_v dt
\]

\[
\Delta u = c_v dt \\
\Delta h = c_p dt
\]

Also \( c_p - c_v = R \)

Couple \( du = c_v dt \) and \( P = \frac{RT}{V} \)

\[
Tds = c_v dt + \frac{RT}{V} dv \\
\]

\[
\frac{ds}{T} = \frac{c_v dt}{T} + R \frac{dv}{V}
\]

Result #1
Integrate between states \( \lambda \) and \( \lambda_* \):

\[
S_2 - S_1 = \int_{1}^{2} C_v \frac{dT}{T} + R \ln \frac{T_2}{T_1}
\]

If \( C_v = \text{const} \):

\[
S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}
\]

Couple \( dh = C_p \, dT \) and \( nV = RT \) with \( \mathbb{H} \):

\[
Tds = C_p \, dT - \frac{RT}{\rho} \, dp
\]

\[
ds = C_p \frac{dT}{T} - \frac{R \, dp}{\rho}
\]

\[
S_2 - S_1 = \int_{1}^{2} C_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}
\]

If \( C_p = \text{const} \):

\[
S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}
\]

Real gas \( \text{cost } C_p \).

What if \( C_p = C_p(T) \)?

Let

\[
S^{\circ}_{T_2} - S^{\circ}_{T_1} = \int_{T_1}^{T_2} \frac{C_p \, dT}{T}
\]

and tabulate \( S^{\circ}_T \) vs \( T \).

Then

\[
S_2 - S_1 = S^{\circ}_{T_2} - S^{\circ}_{T_1} - R \ln \frac{P_2}{P_1}
\]

Look at Table A-17.

Calculate change in entropy for air:

\( T_1 = 500 \text{K} \), \( T_2 = 1200 \text{K} \),

\( P_1 = 100 \text{ kPa} \), \( P_2 = 200 \text{ kPa} \).
a) if $C_p = \text{const.}$ we have $T_{avg} = 850K$

$$C_p = \frac{1}{11} \frac{kJ}{kg-K}$$

$$S_2 - S_1 = \frac{1}{11} \frac{kJ}{kg-K} \ln \frac{1200}{500} - 0.287 \frac{kJ}{kg-K} \ln \frac{200}{100}$$

$$= 0.97177 - 0.19893$$

$$= 0.7728 \frac{kJ}{kg-K}$$

b) if $C_p = C_p(T) - \text{use table}$

$$S_2 - S_1 = S_{T_2}^o - S_{T_1}^o - R \ln \frac{P_2}{P_1}$$

$$= 3.12888 - 2.21982 - 0.287 \ln (2)$$

$$= 0.959360 - 0.198933$$

$$= 0.76043 \frac{kJ}{kg-K}$$

Consider ideal gas in an isentropic process

$$S = c_{v} s +$$

$$\Rightarrow ds = 0$$

$$\int ds = 0 = \int_{1}^{2} c_v \frac{dT}{T} + R \ln \frac{T_2}{T_1}$$

$$\int ds = 0 = \int_{1}^{2} C_p \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

if $C_p = \text{const.}$

$$c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} = 0$$

$$C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0$$
Manipulating, 

\[
\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{V_2}{V_1}\right)^{1-k}
\]

Ideal gas
Isentropic
\(C_p, C_v = \text{const}\)

\[k = \gamma = \frac{C_p}{C_v}\]

\[P_0 e^{-k} = \text{const} \quad !\]

Ideal gas + isentropic + const \(C_p = \text{polytropic} \quad \gamma = \frac{n}{n-1}\)

\[
\ln \frac{P_2}{P_1} = (S_{T_2} - S_{T_1})
\]

\[P_2/P_1 = \exp \left(\frac{S_{T_2} - S_{T_1}}{R}\right) = \exp \left(\frac{S_{T_2}}{R}\right) / \exp \left(\frac{S_{T_1}}{R}\right) = f(T_2) / f(T_1)
\]

Define \(P_r\):

\[
\frac{P_2}{P_1} = \frac{P_{r_2}(T_2)}{P_{r_1}(T_1)} \quad P_r = \exp \left(\frac{S_{T_1}}{R}\right)
\]

Find \(T_2\) for isentropic compression

of AIR from 300K 100 kPa to 500 kPa.
a) \( \frac{d P}{d s} = c \equiv c_{st} \) (let \( \gamma \cdot \frac{v_s}{c_s} = 1.4 \))

\[
T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}}
\]

\[
= 300 \left( \frac{500}{1000} \right)^{0.4} = 475 K
\]

b) if \( C_p = C_p(T) \)

\[
\frac{P_{v2}}{P_1} = \frac{P_2}{P_1} = 1.386 \times \frac{500}{1000} = 0.693
\]

\( f(T_1) \) only

Go back to table, find \( T_2 \) via \( P_{v2} \)

Interpolate between 470K and 480K.