A Direct Transformation Matrices Method for Solution of Multi-Dimensional Inverse Heat conduction Problems

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Introduction

In the present paper a new scheme is presented for inverse heat conduction problems with constant thermophysical properties. The scheme combines the Sequential Function Specification (SFS) method of Beck and the Dual Reciprocity Boundary Element Method (DRBEM). In this scheme the unknown boundary condition is estimated sequentially by using two transformation matrices. One of the matrices takes into account the initial condition, while the other incorporates the measured temperature data. The matrices defined in the direct heat conduction calculations by the Dual Reciprocity Boundary Element Method are used as a basis for the definition of the transformation matrices, and the mathematical derivations for the inverse estimation are in accordance with the Sequential Function Specification of Beck. Contrary to the SFS method, however, this scheme does not require calculation of the sensitivity coefficients; nor does it require an initial guess for the unknown boundary condition as it performs the estimation. That is, once the transformation matrices have been determined, using the string of temperature data, the unknown heat flux components are determined simply by performing two matrix multiplications. A two-dimensional problem with unknown heat flux components, both temporally and spatially, is considered as a test case. In order to compare the speed and the accuracy of the method with the existing SFS method, the exact analytical solution of a simulated test case is utilized. The results for the considered test cases show that for the same order of accuracy the computational speed may be increased by factor of fifty (!) as compared to the classical Sequential Function Specification Method.

Inverse solution

The unknown heat flux is estimated sequentially in which it is assumed that the problem is solved up to t_{M-1}, i.e., the unknown heat flux components and the complete temperature distribution are known up to this time. The unknown heat flux is estimated using the following equation

$$\left[\mathbf{q}^{M}\right]=[\mathbf{C}][\mathbf{Y}]+[\mathbf{D}]\left[\mathbf{T}^{M-1}\right], \tag{1}$$

where

$$[\mathbf{Y}]^{T}=[\mathbf{Y}(M),\mathbf{Y}(M+1),......\mathbf{Y}(M+r-1)]. \tag{2}$$

Direct solution

Dual Reciprocity Boundary Element method is used to perform direct solution. As the unknown heat flux is estimated, the direct solution is performed using the following equation

$$\left[\mathbf{T}^{M}\right]=[\mathbf{A}]\left[\mathbf{q}^{M}\right]+[\mathbf{B}]\left[\mathbf{T}^{M-1}\right], \tag{3}$$

where $\mathbf{T}^{M}$ is used as initial condition for the next time step.

Cleary the algorithm, which contains the direct and inverse solutions, is performed via four matrix multiplications.

Equations(1) and(3) are obtained by assuming adiabatic boundary condition for the inactive surface. In the more general cases as will be shown in the paper, some other matrix multiplication may be required to take into account boundary condition of the inactive surface.

Numerical results

A simulated experiment was used to show efficiency and accuracy of the method. The problem has been formulated based on an analytic solution, which has already used by some other researchers [1,2]. The geometry and boundary conditions of the problem is demonstrated in fig-1. The problem has been solved both with SFSM of Beck and the presented method (TMM), while DRBEM has been used as computational tool for direct discretization in both of the methods.

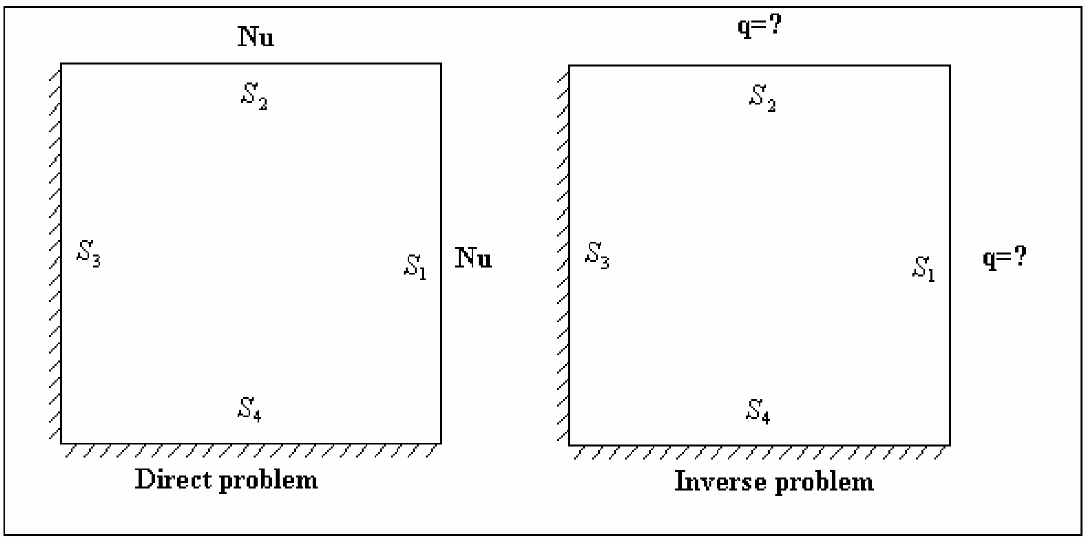

Fig 1: The geometry and boundary conditions of the test case

The computational time for the two methods for 100 time steps and for different number of future time steps is compared in table-1. The results show significant reduction in computational time.

r	5	10	15	20
TMM	0.02	0.02	0.02	0.02
SFSM	0.22	0.55	0.93	1.04

Table 1: comparison of computational time of classical SFSM and TMM for 100 time steps (seconds)

References

[1] K. Kurpitz, A.J. Nowak, Inverse thermal problems, Computational mechanics publications, Souhampton, USA(1995)

[2] Krishna M. Singh, Masataka Tanaka, Dual reciprocity boundary element analysis of inverse heat conduction problems, Comput. Methods Appl Mech. Engrg. 190 (2001) 5283-5295